

# Power System Operations

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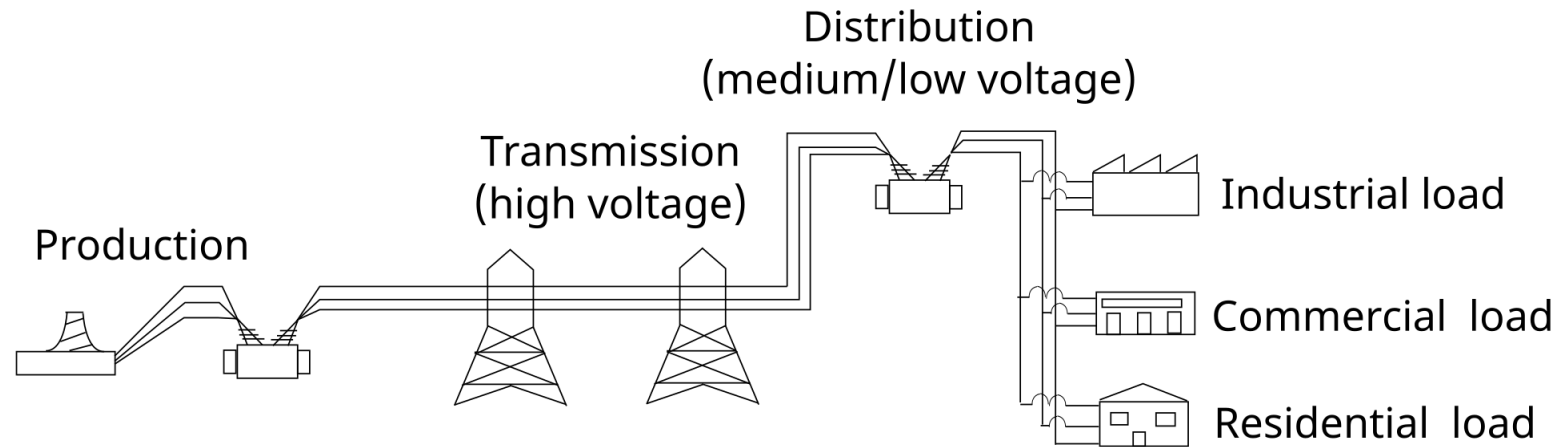
Source: chapter 3.1, Papavasiliou [1]

# Outline

- Production
- Transmission and distribution
- Consumption
- Actors
- Uncertainty and reserve
- Stages of decision making

# Production

# Power system supply chain



- Different components of power systems:
  - Production
  - Transmission and distribution
  - Consumption

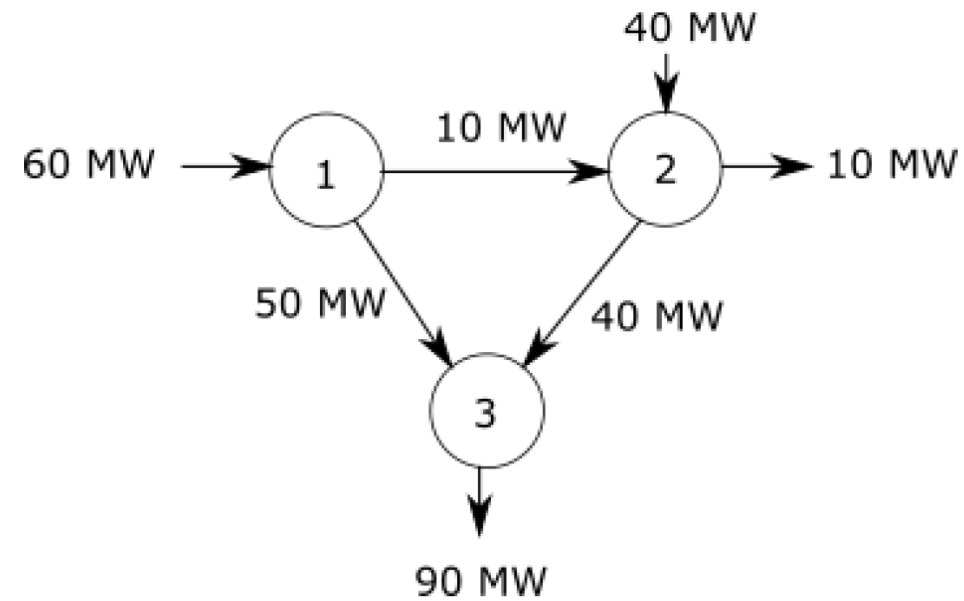
# Production

- Non-renewable energy sources:
  - Fossil fuels (coal, oil, natural gas)
  - Nuclear
- Renewable energy sources:
  - Hydroelectric (run of river, dams, pumped storage)
  - Geothermal
  - Wind
  - Solar
  - Biomass
  - Other (wave, tidal)

# Units of measurement

- Energy is measured in megawatt hours (denoted MWh)
- Power is energy per unit time:
  - Rate of energy production
  - Rate of energy consumption
  - Energy flow
- Power is measured in MW

# The pool analogy



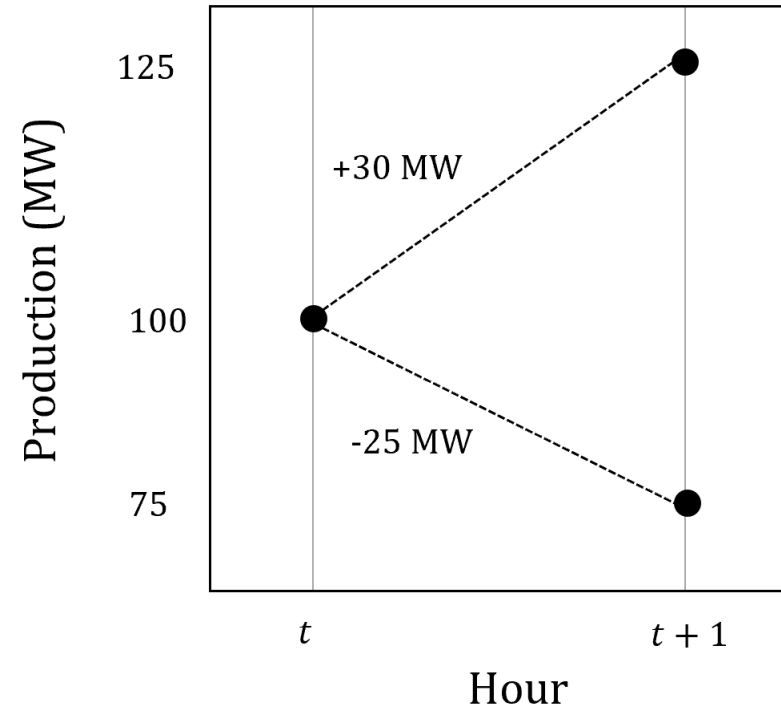
Who is supplying the load in node 3;

# Production constraints

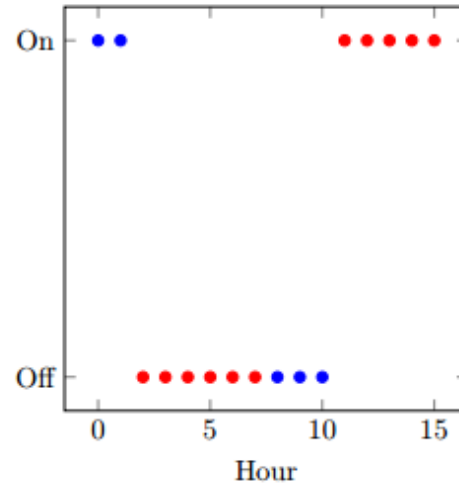
- Fossil fuel constraints:
  - Min/max production
  - Ramp constraints
  - Min up/down times
  
- Hydro constraints:
  - Max production
  - Max storage



# Ramp constraints



# Min up/down time constraints



- Min up time: 5 hours
- Min down time: 6 hours
- Red dots: constrained decisions
- Blue dots: free decisions

# Variable and marginal cost

**Variable/operating/fuel cost:** cost that depends on amount of output

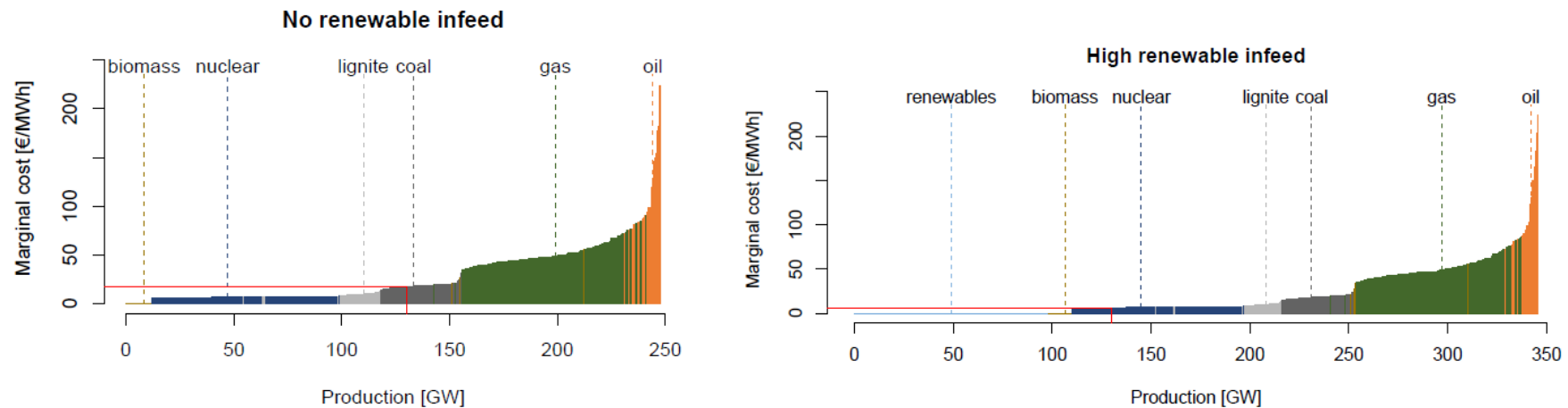
- Measured in \$/h
- Hourly cost of producing a certain amount of power

**Marginal cost:** derivative of fuel cost with respect to output

- Measured in \$/MWh
- Increase in fuel cost if an additional MW of power were produced/saving if one less MW is produced

# Merit order curve

**Merit order curve:** (increasing) system marginal cost curve



What is the impact of renewable energy on the system?

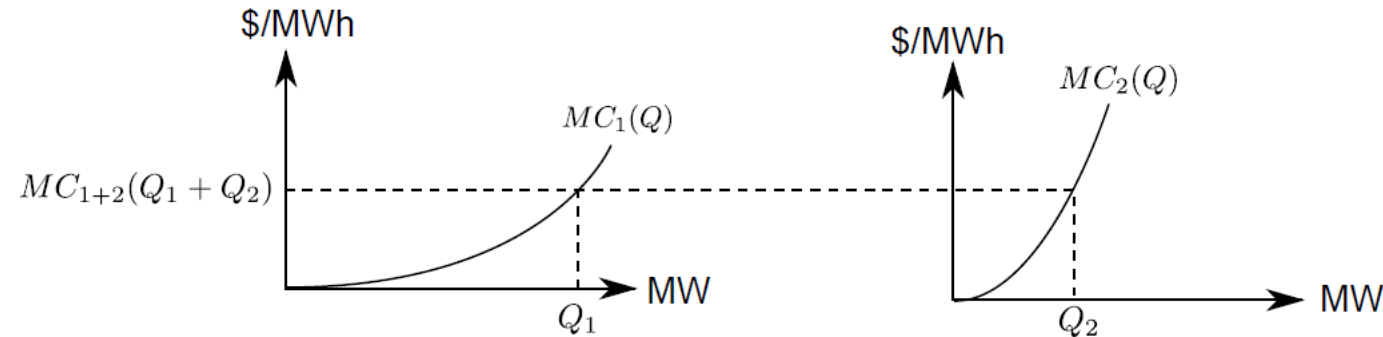
# Marginal cost range

If variable cost is non-differentiable, define

- **Left-hand marginal cost:** left-hand side derivative of marginal cost
- **Right-hand marginal cost:** right-hand side derivative of marginal cost (**when is it infinite?**)
- **Marginal cost range:** set of values between and including left- and right-hand marginal cost

# Horizontal summation of marginal costs

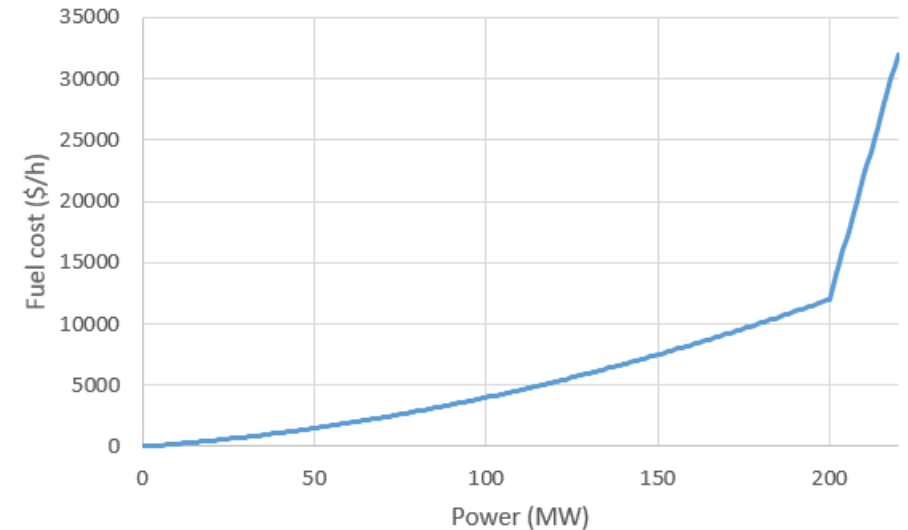
Aggregate marginal cost is obtained by horizontal summation



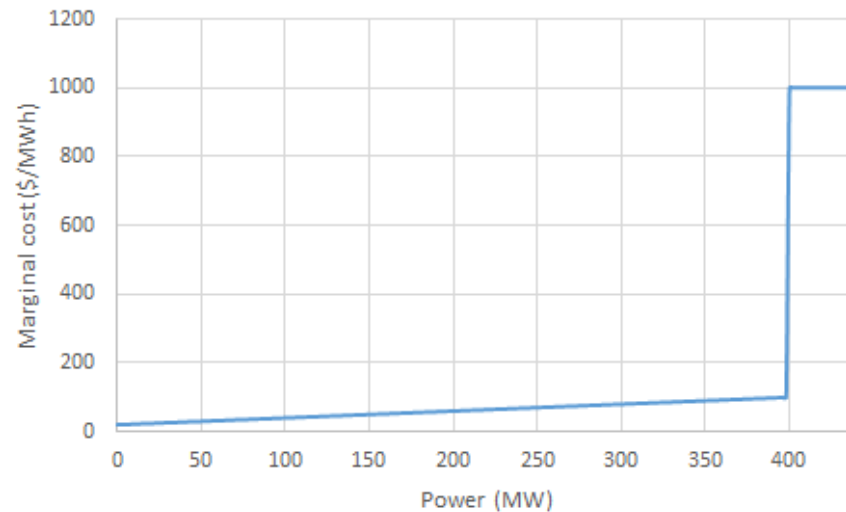
# Example

Consider quadratic fuel cost with:

- $MC(0 \text{ MW}) = 20 \text{ \$/MWh}$
- $MC(200 \text{ MW}) = 100 \text{ \$/MWh}$
- $MC(Q) = 1000 \text{ \$/MWh}, 200 \text{ MW} \leq Q \leq 220 \text{ MW}$
- $VC(0 \text{ MW}) = 0 \text{ \$/h}$



# Summation of marginal cost curves



This is the marginal cost of  $n$  generators from previous slide,  $n = ?$



# Investment cost

- Fixed/investment cost: cost that is independent of output
  - **Overnight cost** (\$/kW): cost that needs to be paid upfront per kW of investment
  - **Annualized fixed cost** (\$/kW<sub>y</sub>): cost that needs to be paid per year per kW of investment
  - **Hourly fixed cost** (\$/MWh): cost that needs to be paid per hour per MW of investment

# Conversion of investment cost

Denote:

- $T$  (years): investment lifetime
- $r$ : interest rate

**Annualized fixed cost  $FC$  (\$/kW<sub>y</sub>) given *annual discounting***

$$FC = \frac{r \cdot OC}{1 - 1/(1+r)^T}$$

**Annualized fixed cost  $FC$  (\$/kW<sub>y</sub>) given *continuous discounting***

$$FC = \frac{r \cdot OC}{1 - e^{-rT}}$$

**Hourly fixed cost (\$/MWh):** divide annualized fixed cost by 8.76 (why 8.76?)

# Example

- Gas turbine lifetime: 25 years
- Coal generator lifetime: 45 years
- Continuous discounting with interest rate  $r = 12\%$

	<i>OC</i> (\$/kW)	<i>FC</i> (\$/kW <sub>y</sub> )	<i>FC</i> (\$/MWh)
Gas turbine	400	50.5	5.8
Coal	1200	144.7	16.5

# Average cost

## **Average cost:** total cost per unit of output

- Definition of average cost generalizes to the case of an *industry* that produces  $Q$  at minimum cost
- Economies of scale are realized when average cost decreases
- Average cost influences whether an industry is a **natural monopoly** or not

# Intersection of average cost curve and marginal cost curve

- By definition:

$$AC(Q) = \frac{TC(Q)}{Q} = \frac{FC + VC(Q)}{Q}$$

- The minimum of the average cost curve is characterized by the following condition:

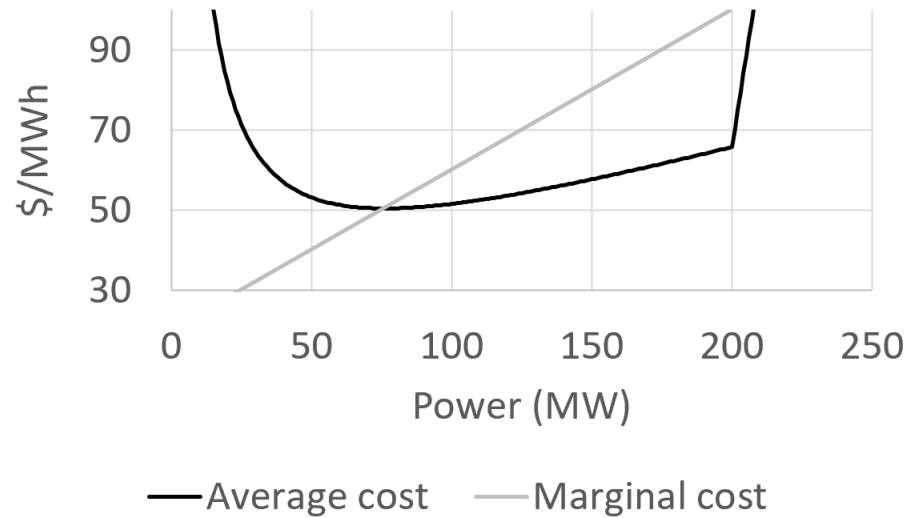
$$AC'(Q^*) = 0 \Rightarrow$$

$$\frac{MC(Q^*) \cdot Q^* - (FC + VC(Q^*))}{(Q^*)^2} = 0 \Rightarrow$$

$$MC(Q^*) = \frac{FC + VC(Q^*)}{Q^*} \Rightarrow$$

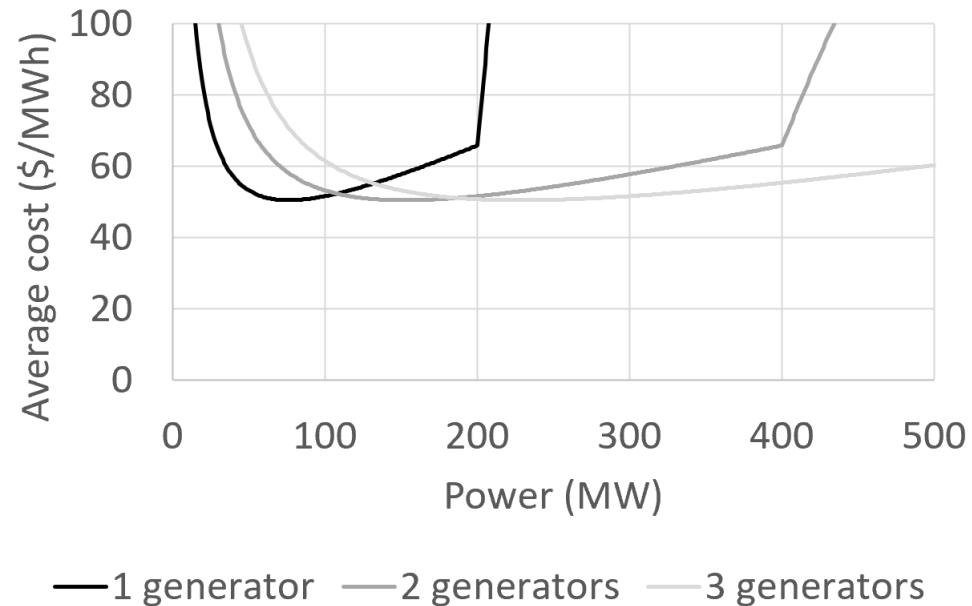
$$MC(Q^*) = AC(Q^*)$$

# Average cost curve: single generator



Why is there a jump in average cost at  $Q = 0$  MW?

# Average cost curve: multiple generators



Average cost at unit capacity (200 – 220 MW) is lower for  $n = 3$  generators

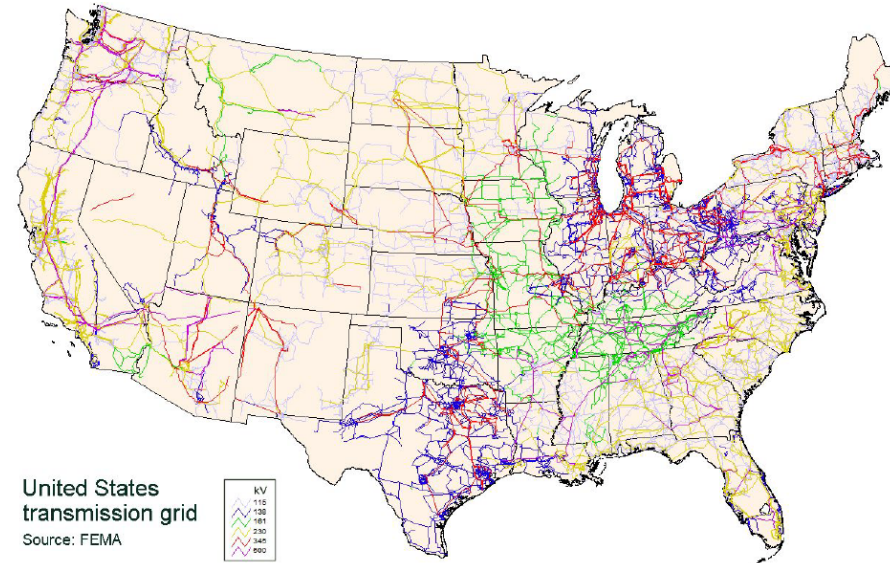
# Minimum efficient scale and natural monopoly

- The point at which average cost is minimized is called **minimum efficient scale**
- If the minimum efficient scale is comparable to the level of demand in the system, this is an indication of a **natural monopoly**
- Natural monopolies require government intervention/regulation
- Natural monopolies tend to emerge in industries where fixed cost dominates variable cost
- Electric power systems exhibited natural monopoly characteristics until recently



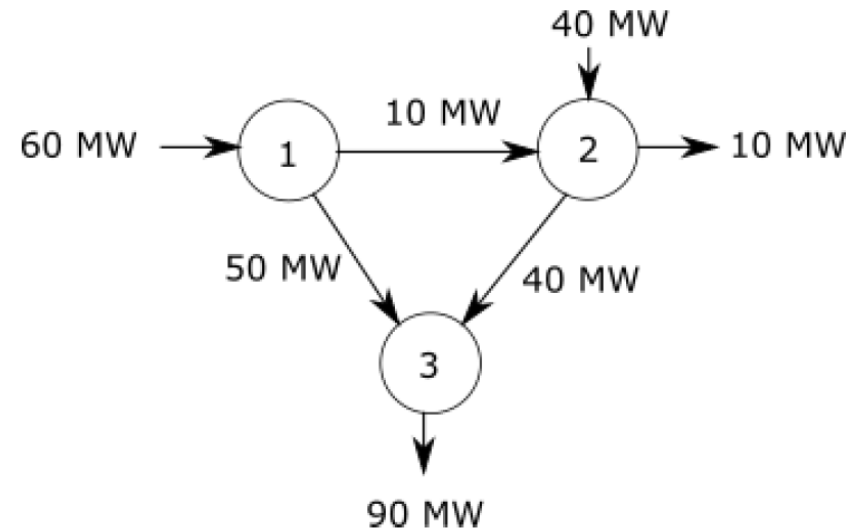
# Transmission and distribution

# Transmission and distribution



- Transmission: higher voltage, lower losses
- Distribution: lower voltage, higher losses
- Transformers reduce voltage at the interface

# Power balance

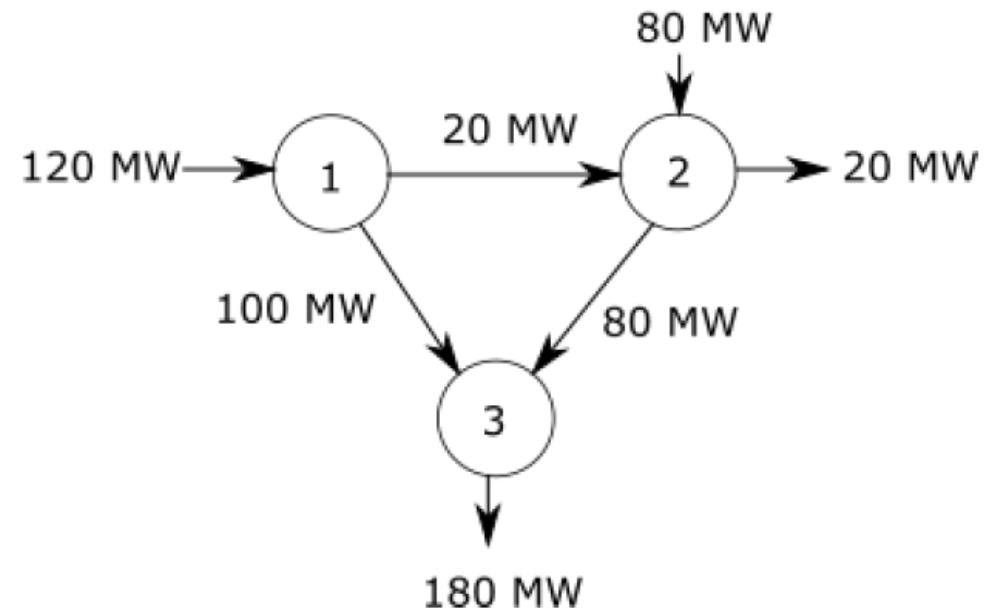
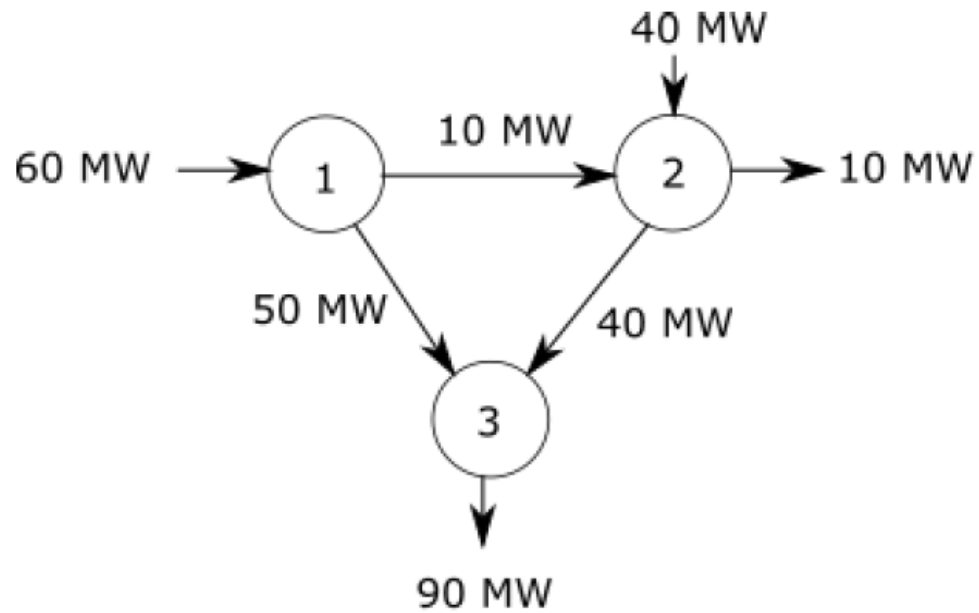


- **Buses:** nodes of the transmission network
- **Branches/lines:** edges of the transmission network
- Power balance at each bus (same as transportation models)
- Physical intuition: electricity is «lazy»

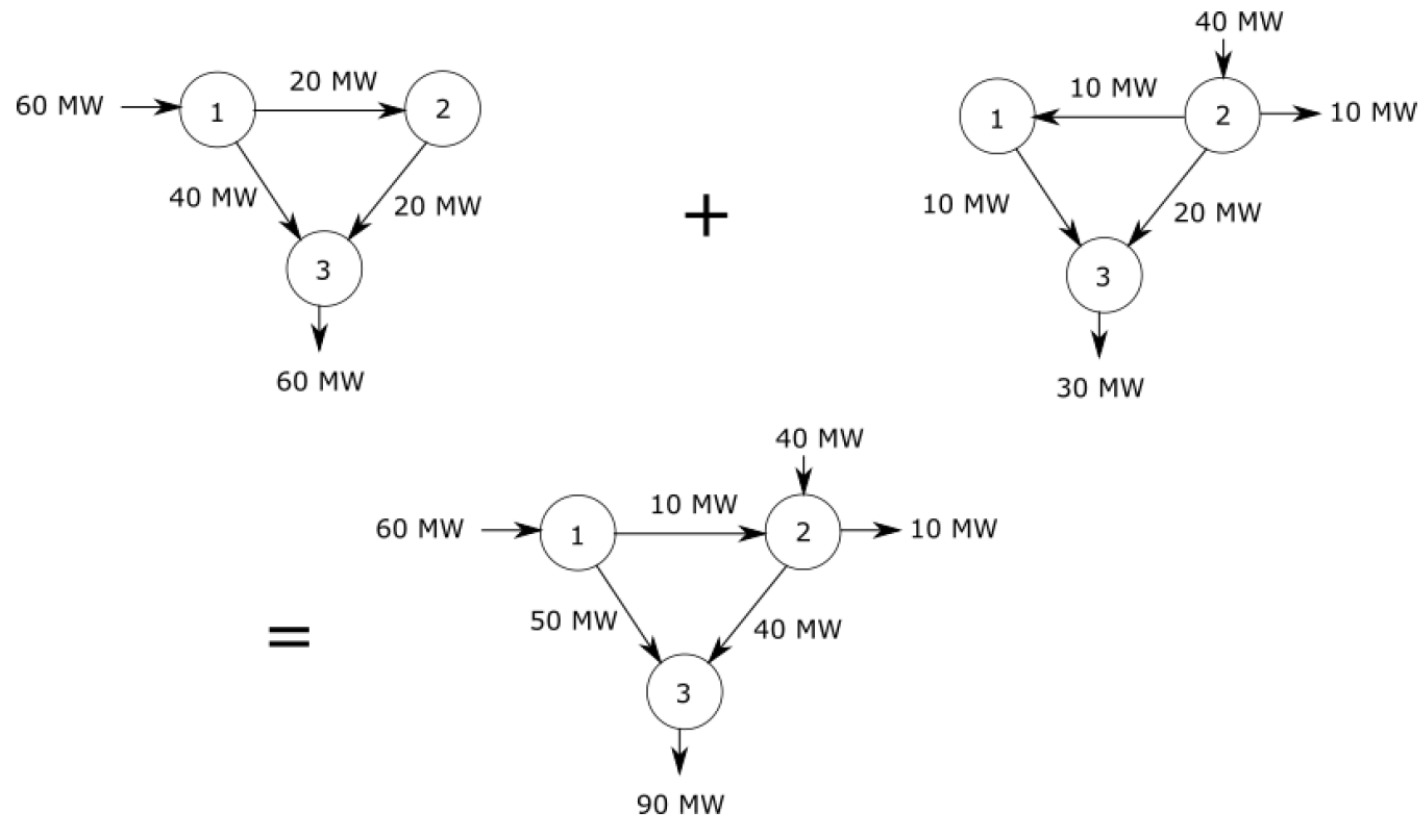
# Power flow equations

- Power networks are more complex than transportation networks
- **Kirchhoff's laws:** physical laws that govern flow of electricity in circuits, can be used to derive power flow equations
- **Power flow equations** determine a mapping  $f = P(r)$  of power injections  $r$  in buses to power flows  $f$  on lines
- **Direct current (DC) power flow equations:** approximation of power flow equations by a *linear* mapping

# Proportionality of power flows



# Additivity of power flows



# Consumption

# Valuation and benefit

**Consumer benefit:** benefit that depends on amount of consumption

- Measured in \$/h
- Hourly benefit of consuming a certain amount of power

• **Marginal benefit/valuation:** derivative of benefit with respect to consumption

- Measured in \$/MWh
- **Willingness to pay** (why?)

What is the supply-side analog of consumer benefit? of valuation?

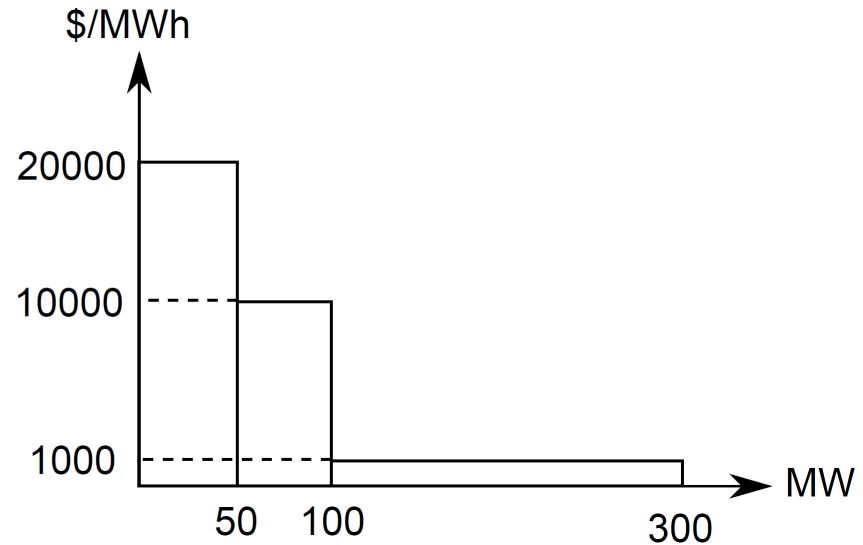
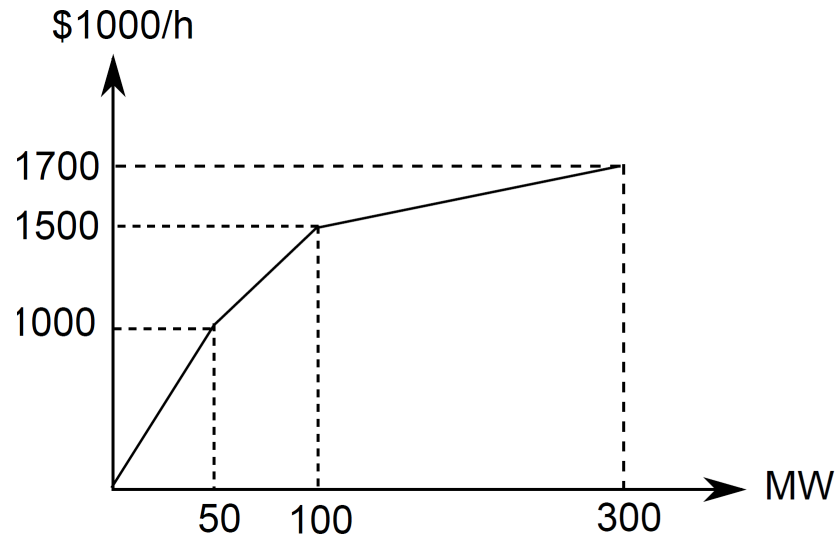


# Inverse demand/marginal benefit function

**Inverse demand/marginal benefit function:** mapping of power consumption  $Q$  to marginal benefit  $MB(Q)$

Do we expect an inverse demand function to be increasing/decreasing?

# Illustration of marginal benefit function



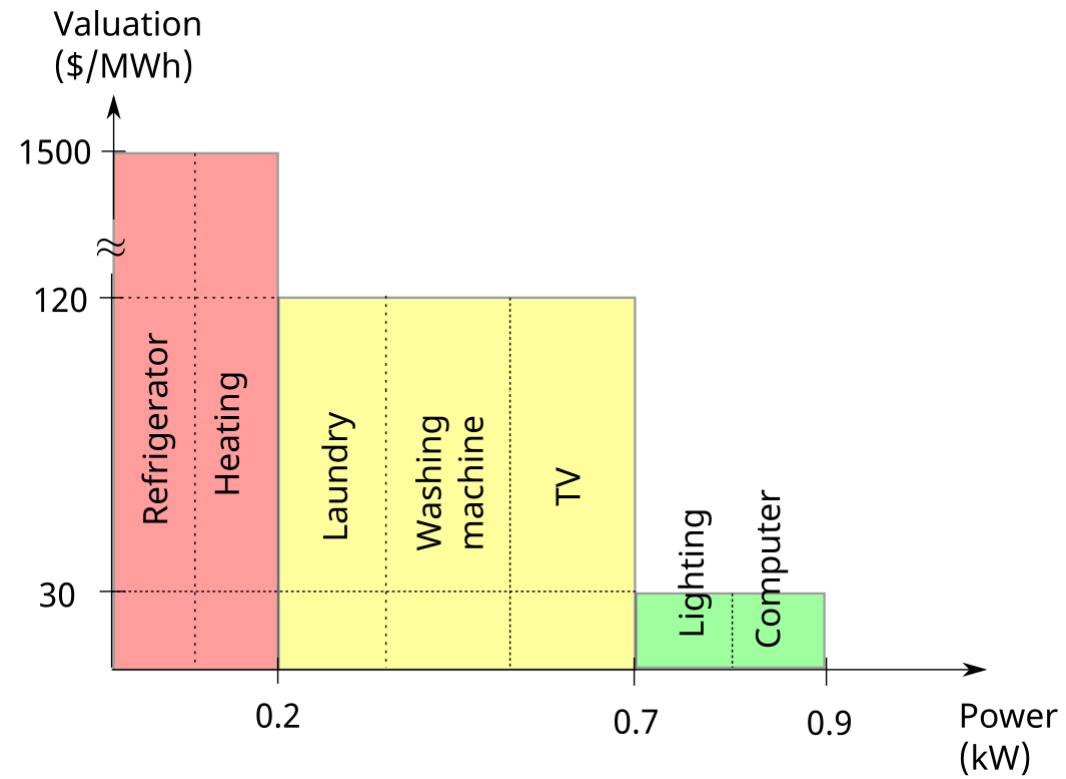
Which graph corresponds to consumer benefit? inverse demand function?

# Example: household

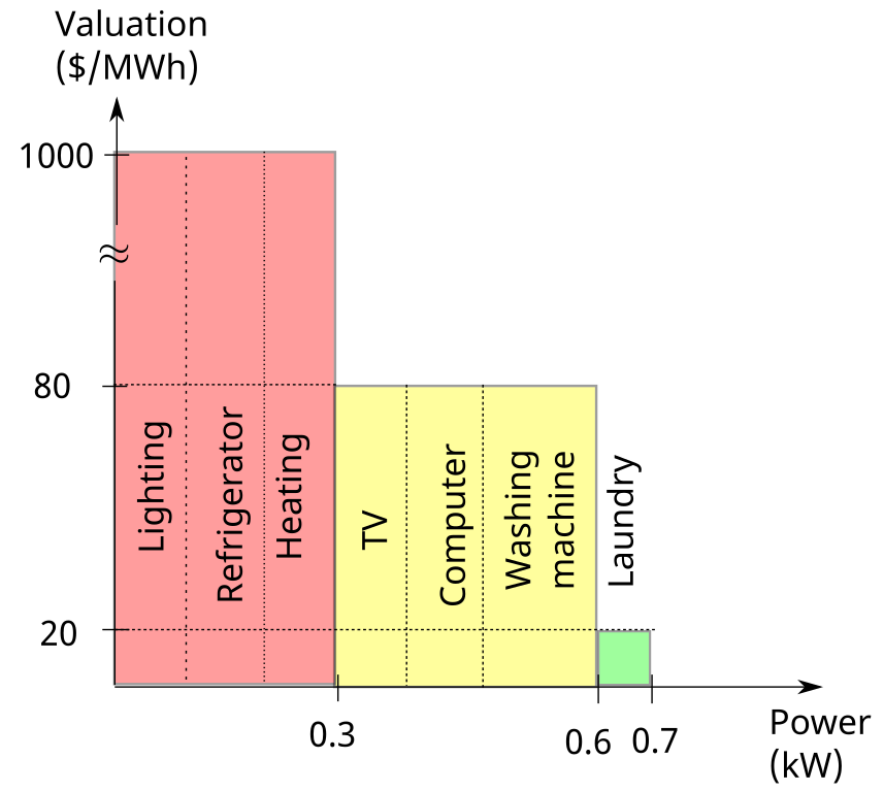
Tranche	Demand morning (kW)	Valuation morning (\$/MWh)	Demand evening (kW)	Valuation evening (\$/MWh)
Inflexible	0.2	1500	0.3	1000
Medium	0.5	120	0.3	80
Flexible	0.2	30	0.1	20

Devices can move from tranche to tranche  
(which devices could belong to the inflexible tranche?)

# Morning



# Evening



# Demand function

**Demand function**  $Q(v)$ : inverse mapping of inverse demand function

Maps price of power  $v$  to quantity  $Q$  that would be procured

# Elasticity of demand

**Elasticity:** sensitivity of demand  $Q(v)$  to changes in price  $v$ :

$$\varepsilon = \frac{dQ/dv}{Q/v}$$

Steep inverse demand function  $\Leftrightarrow$  flat demand function  $\Leftrightarrow$  low  $\varepsilon \Leftrightarrow$  inelastic (insensitive) demand

# Average value of lost load and VOLL

- **Average value of lost load:** long-run average amount of load shed due to random disturbances (failures of generators and lines, forecast errors of renewable resources and load, etc.)
- **Value of lost load (VOLL):** marginal change in average value of lost load due to marginal increase in system capacity, divided by marginal decrease in the amount of shed load
- VOLL useful in capacity expansion planning studies for quantifying marginal benefit of investment in generation capacity



# Example: VOLL

Consider the following demand function:

$$Q(v) = 30000 - 2v$$

Lost value from 1% in service with random rationing

$$\begin{aligned} & \int_{v=0}^{15000} Q(v)dv - \int_{v=0}^{15000} 0.99Q(v)dv = \\ & = 0.01 \cdot \frac{15000 \cdot 30000}{2} = 2.25 \cdot 10^6 \text{ €} \end{aligned}$$

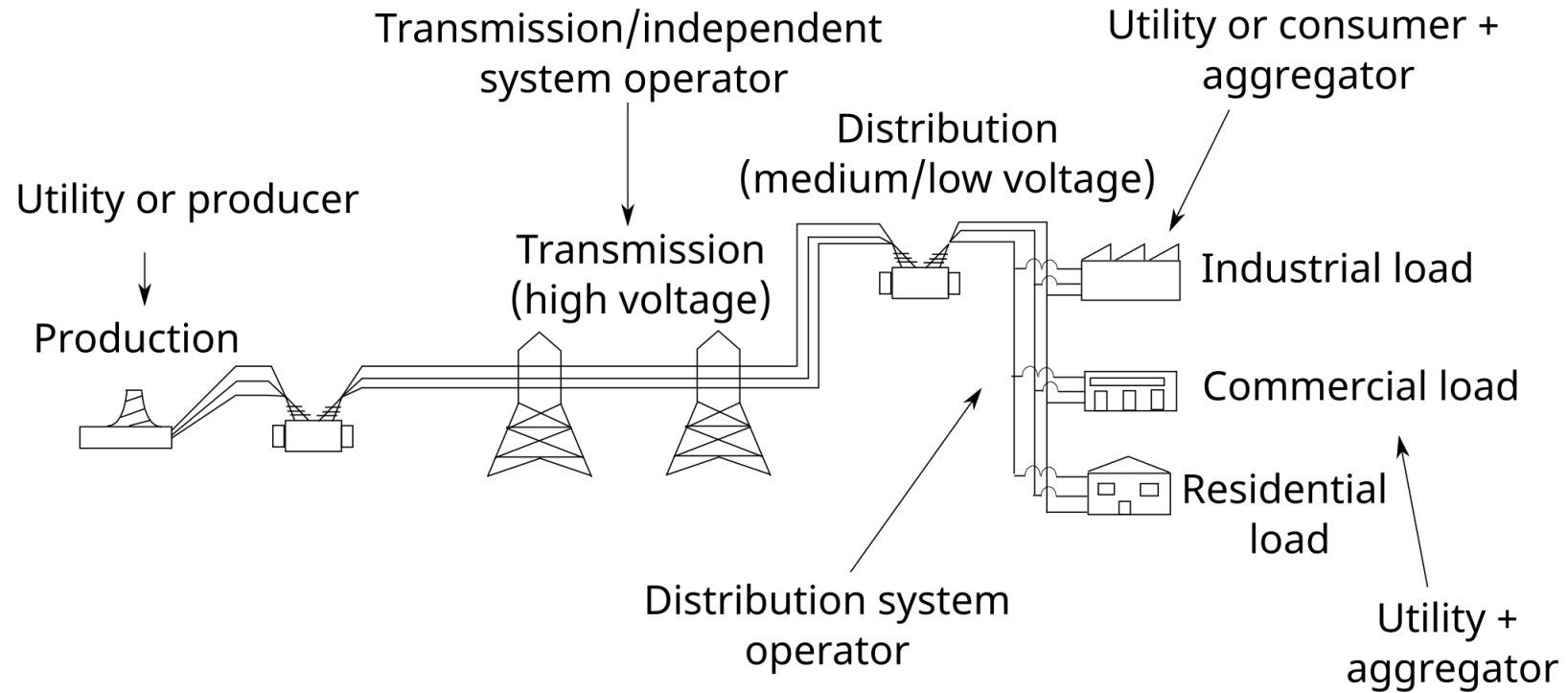
Energy shed from 1% rationing: 300 MWh

$$VOLL = \frac{2250000}{300} = 7500 \frac{\text{€}}{\text{MWh}}$$

More sophisticated computation of VOLL via simulation

# Actors

# Actors



# Uncertainty and reserves

# Uncertainty

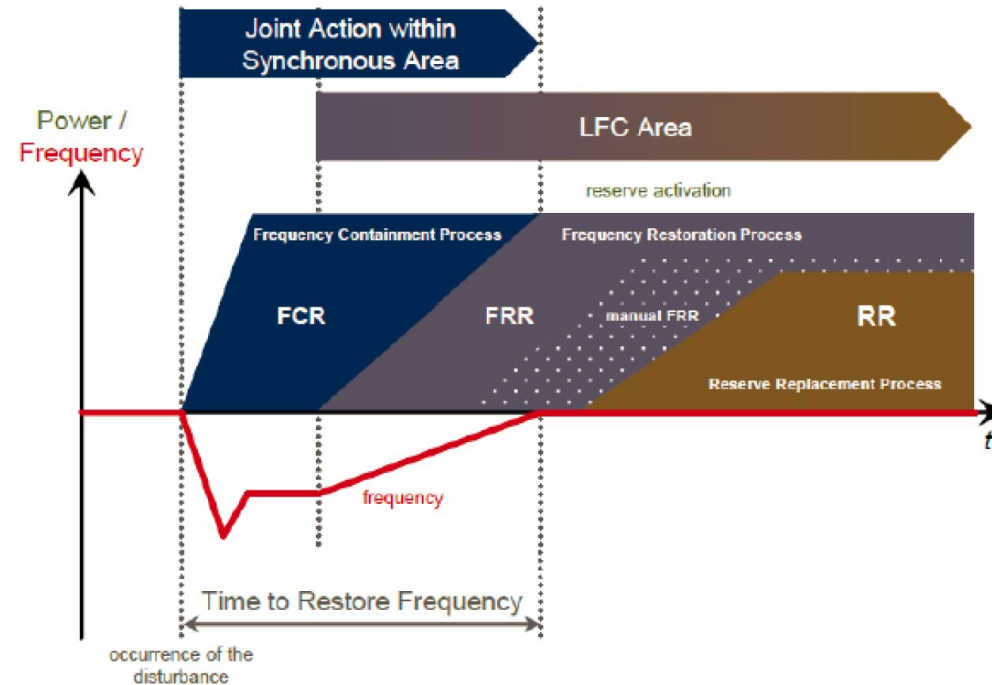
- Rainfall (affects hydro)
- Load forecast errors
- Renewable supply forecast errors
- Generator failures
- Transmission line failures
- Load failures

**Contingency:** failure of any system element (generator, line, transformer, load)

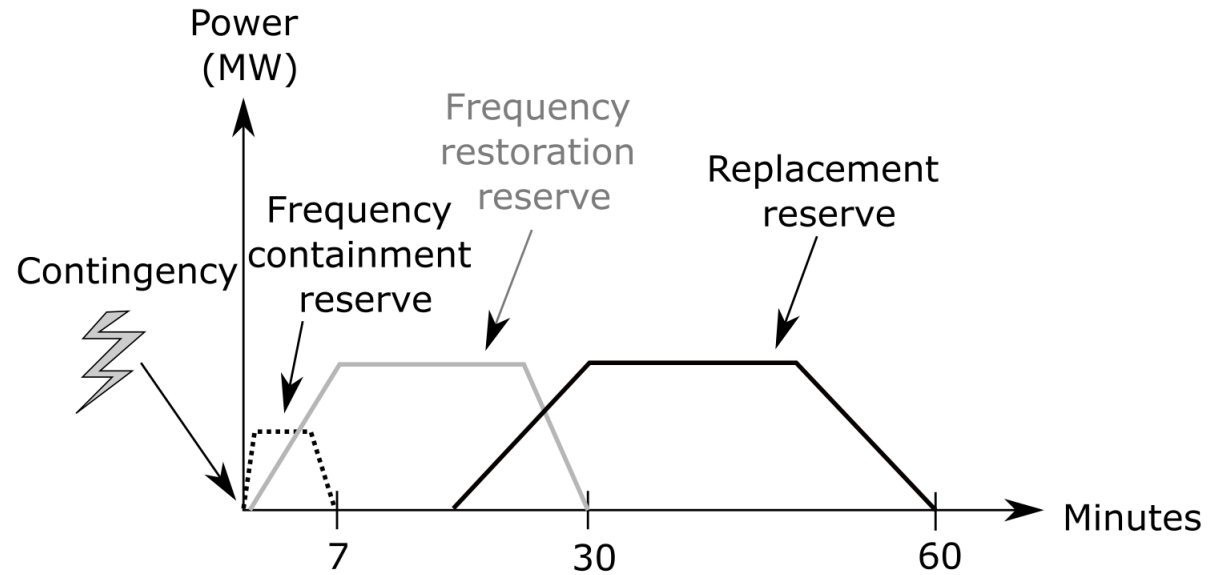
- ❖ Which of these uncertainties are short-term (hours ahead or in real time)?
- ❖ Which of these uncertainties are continuous/discrete?

# Frequency control and restoration

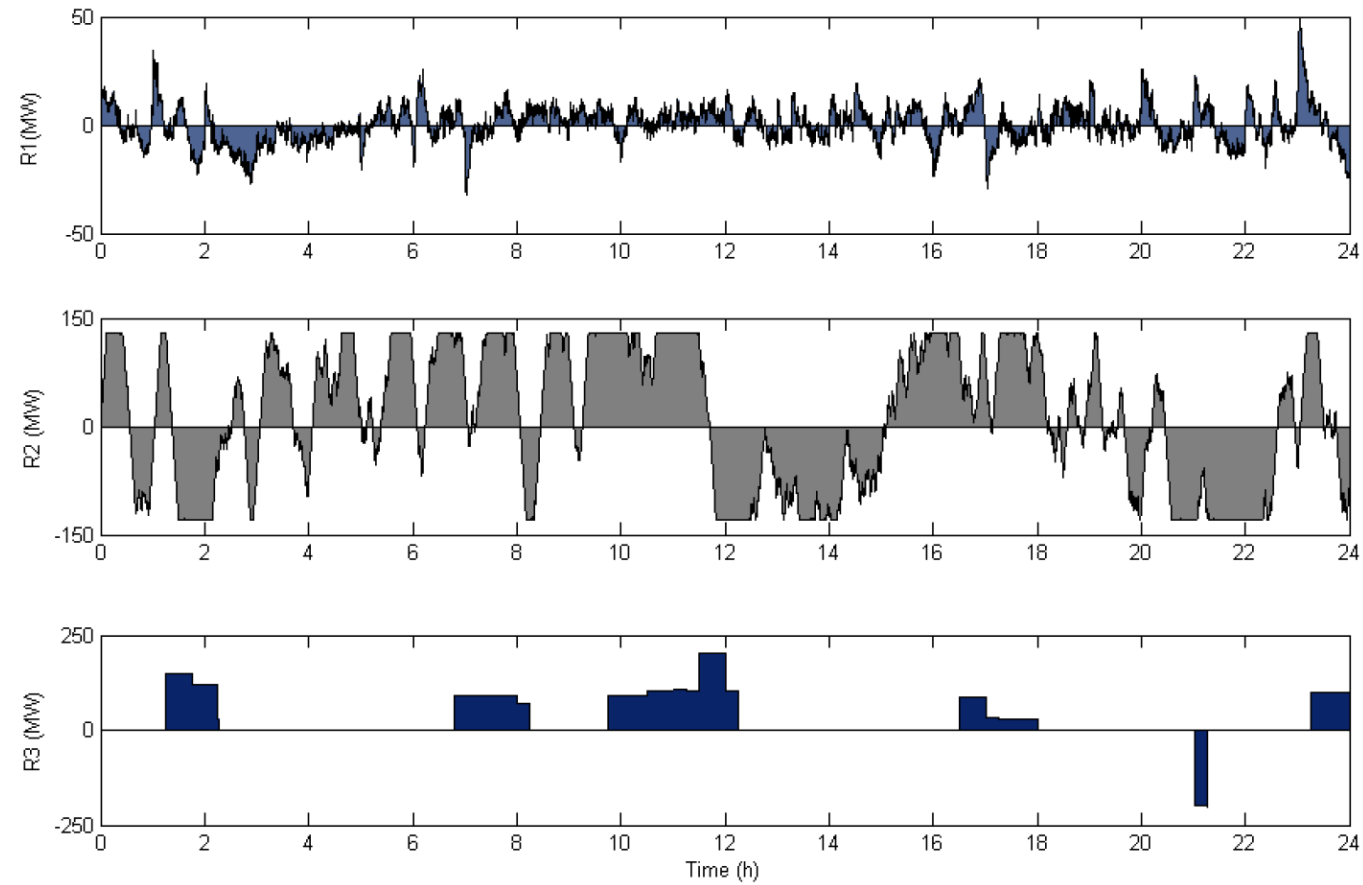
System frequency is an indicator of supply-demand balance



# Sequential activation of reserves



# Reserves in Belgium





# Cost minimization with reserves

Consider  $n$  generators, operating cost  $f_i$ , capacity  $C_i$ , power demand  $D$

$$\min_{p,r} \sum_{i=1}^n f_i(p_i)$$

$$\text{s. t. } p_i + r_i \leq C_i, i = 1, \dots, n$$

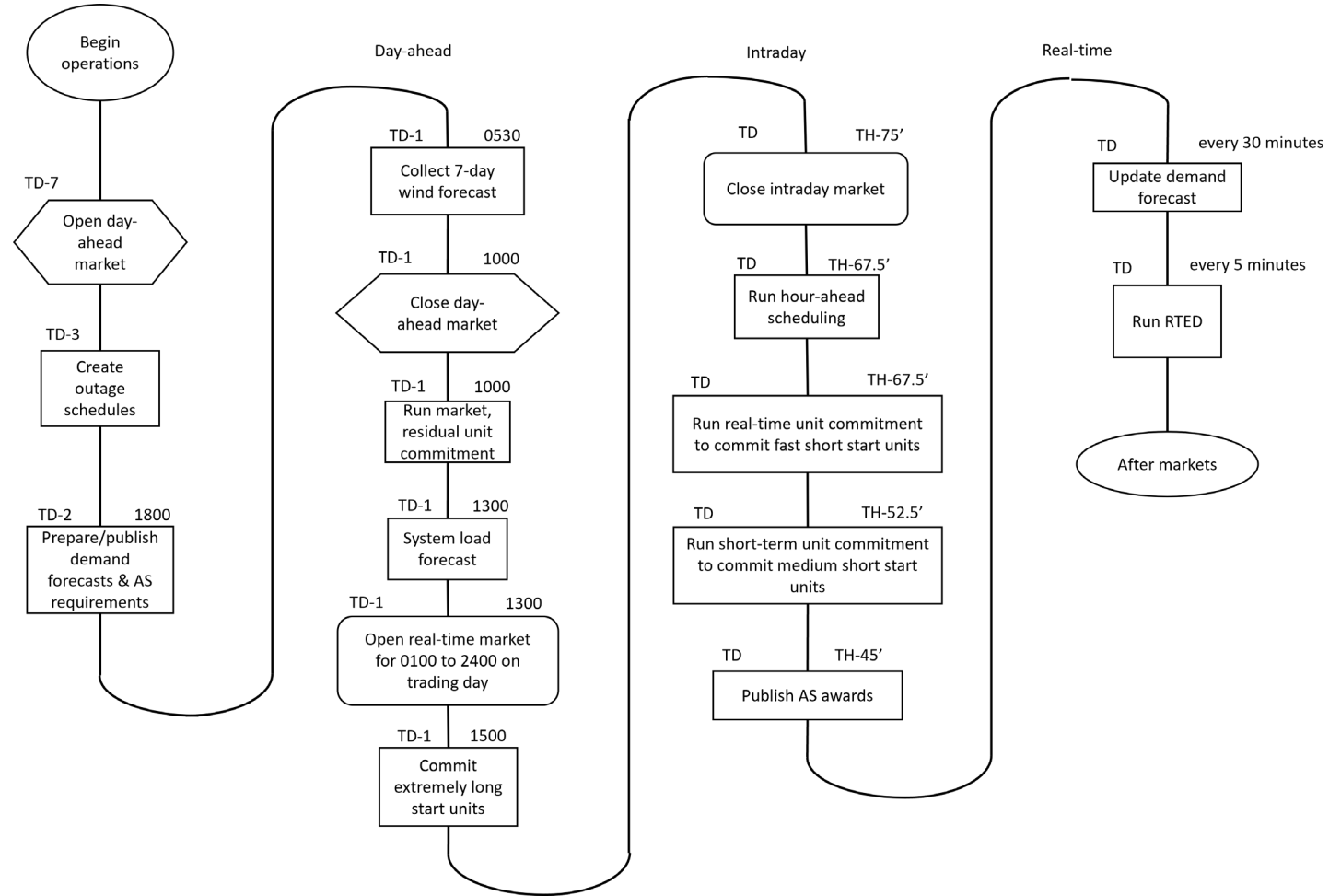
$$\sum_{i=1}^n p_i = D$$

$$\sum_{i=1}^n r_i \geq \max_{i=1,\dots,n} C_i$$

$$p_i, r_i \geq 0$$

# Stages of decision-making

# Flow chart of operations



# Analyzing the flow chart

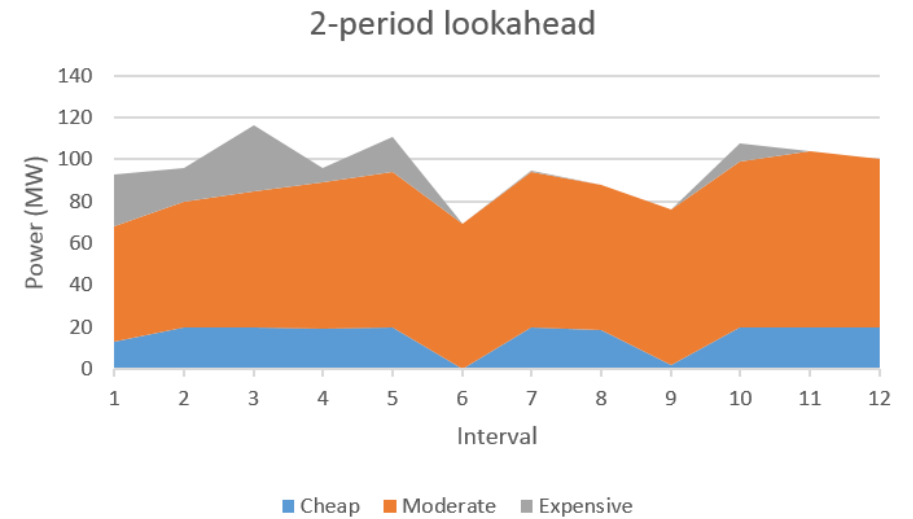
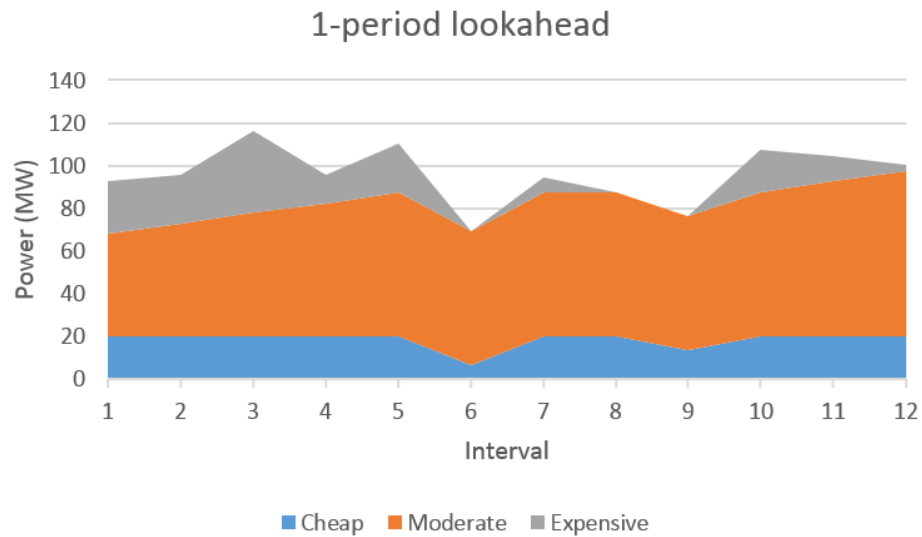
- Which decisions are binding before the day ahead/in the day ahead/in real time?
- What happens if the system operator demand forecast is much higher than traded power in day-ahead market?
- What parts of the supply chain are not actively controlled, according to the flow chart?
- Where would demand response enter in this flow chart?
- How many optimization models are shown in the flow chart?
- What would happen if each optimization model ignored future time periods?

# Example: looking ahead in operations

Consider the following example with three generators:

- Real-time economic dispatch: solved every 5 minutes for the next 5 minutes
- Initial conditions: 50 MW from expensive and 50 MW from moderate
- Demand: Gaussian with mean 100 MW, standard deviation 15 MW

Generator	Marginal cost (\$/MWh)	Max (MW)	Ramp (MW/minute)
Cheap	0	20	$+\infty$
Moderate	10	$+\infty$	1
Expensive	80	$+\infty$	5



- Cost 5-minute lookahead: \$1738
- Cost 10-minute lookahead : \$1406

Why is the second policy doing better?

# References

[1] A. Papavasiliou, Optimization Models in Electricity Markets, Cambridge University Press

<https://www.cambridge.org/highereducation/books/optimization-models-in-electricity-markets/0D2D36891FB5EB6AAC3A4EFC78A8F1D3#overview>