Power Flow

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Source: appendix B, Papavasiliou [1]

Outline

- Graph Laplacian
- Circuits
- The power flow equations
- From net power injections to bus angles
- From bus angles to line flows
- Losses

Graph Laplacian

Incidence matrix

Consider a graph $G = (N, E)$, where N is the set of nodes and E is the set of edges

The **incidence matrix** of the graph is defined as $A = (A_{ij}), i, j \in N$, where

$$A_{ij} = \begin{cases} 1, & \text{if } (i,j) \in E \text{ or } (j,i) \in E \\ 0, & \text{otherwise} \end{cases}$$

Degree matrix

Consider a graph $G = (N, E)$

The **degree matrix** is defined as $D = (D_{ij}), i, j \in N$, where

$$D_{ij} = \begin{cases} d_i, & \text{if } i = j \\ 0, & \text{otherwise} \end{cases}$$

d_i is the degree of node i, which is the number of edges that are incident to the node

For weighted graphs the degree generalizes to the sum of the weights of the edges that are indecent to the node

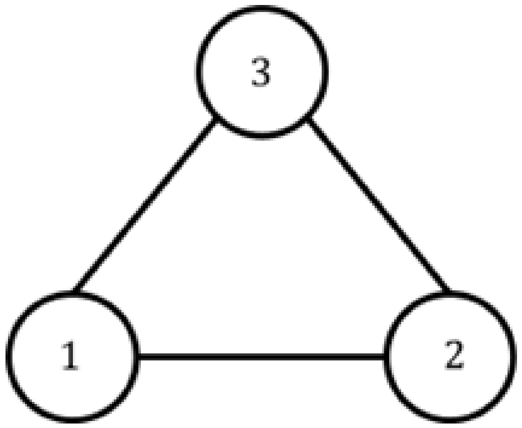
Graph Laplacian

Consider a graph $G = (N, E)$, where A is its incidence matrix and D is its degree matrix

The **Laplacian** of the graph is defined as

$$L = D - A$$

Example: three-node graph



Incidence matrix:

$$A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Example: three-node graph

Degree matrix:

$$D = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

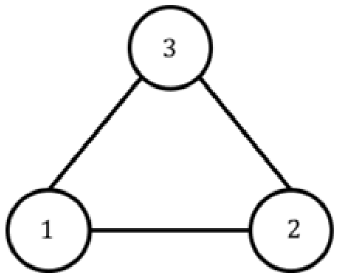
Laplacian:

$$L = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

Laplacian theorem

The multiplicity of the eigenvalue $\lambda = 0$ in the Laplacian of a graph is equal to the number of connected components of the graph

Example: three-node graph



Laplacian:

$$L = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix}$$

Laplacian eigenvalues: $\lambda_1 = 0$, $\lambda_2 = 3$ και $\lambda_3 = 3$

As long as the graph is connected, it has a unique eigenvalue that is equal to zero

Circuits

Circuits

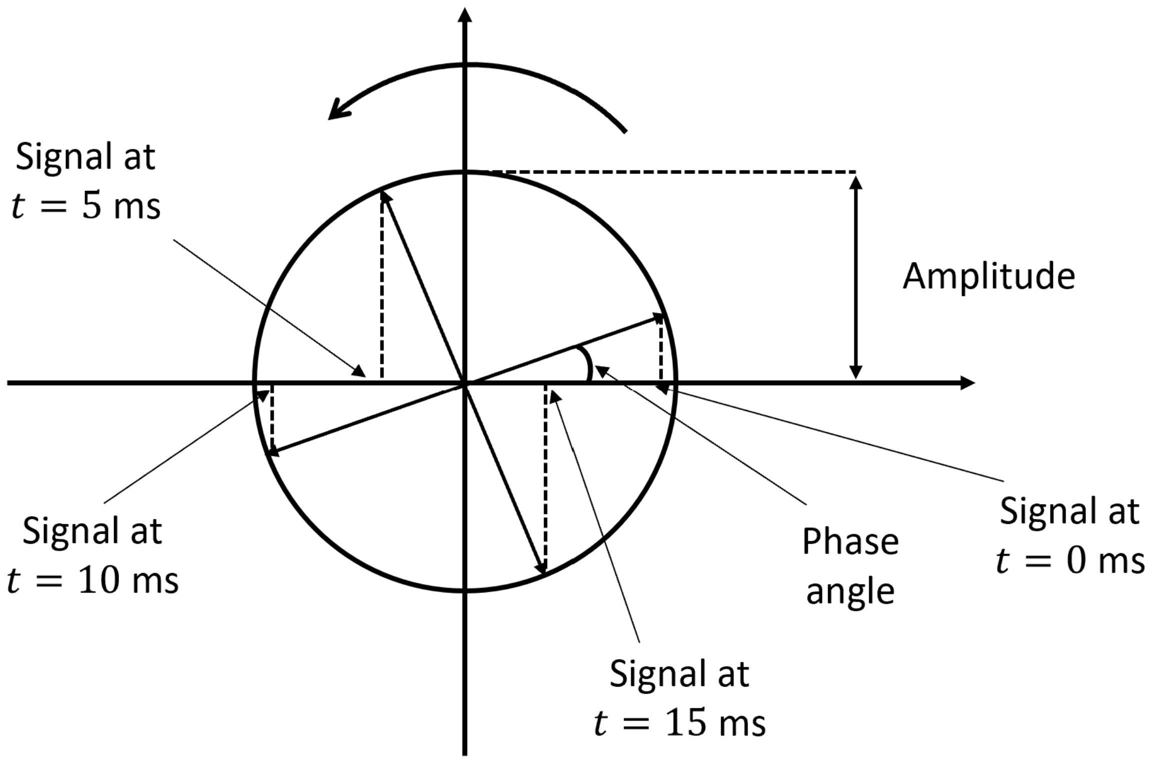
Electric circuits consist of

- **passive elements** (transmission lines, transformers, loads represented as impedances)
- **active elements** that generate or consume power (generators or loads represented as constant consumptions of apparent power)

State of a circuit can be described by the voltage between each node and a reference point, referred to as **ground**

When voltages are known, we know *everything* about the circuit

Representation of a sinusoidal signal as a complex number



Alternating current (AC) electric power systems

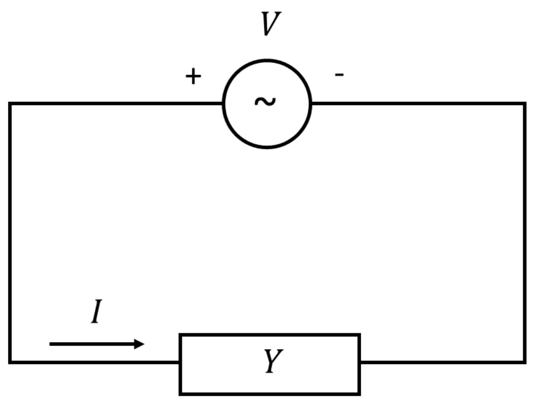
AC power systems have voltage and current that fluctuate sinusoidally (50 Hz in Europe, 60 Hz in USA)

Sinusoidal signals can be described as complex numbers:

- **Amplitude**: magnitude of the complex number
- **Phase angle**: angle of the complex number

The sinusoidal signal can be represented equivalently by its **phasor**, which is characterized by the phase angle and **root mean square** value, which is the amplitude divided by $\sqrt{2}$

Impedance and admittance



$$I = YV, S = VI^*, = P + Qi$$

A passive electric element is characterized by a complex number, the **impedance** Z, or equivalently the **admittance** Y, which is the inverse of impedance

What characterizes a passive element is the fact that current I flowing through the element as a result of voltage V applied on the terminals of the element is

$$I = YV$$

Example: current flowing through a passive element

Consider applying a voltage with an RMS of 230 V on a passive element with $Y = 0.01 - 0.01i\ \Omega^{-1}$

By definition of impedance

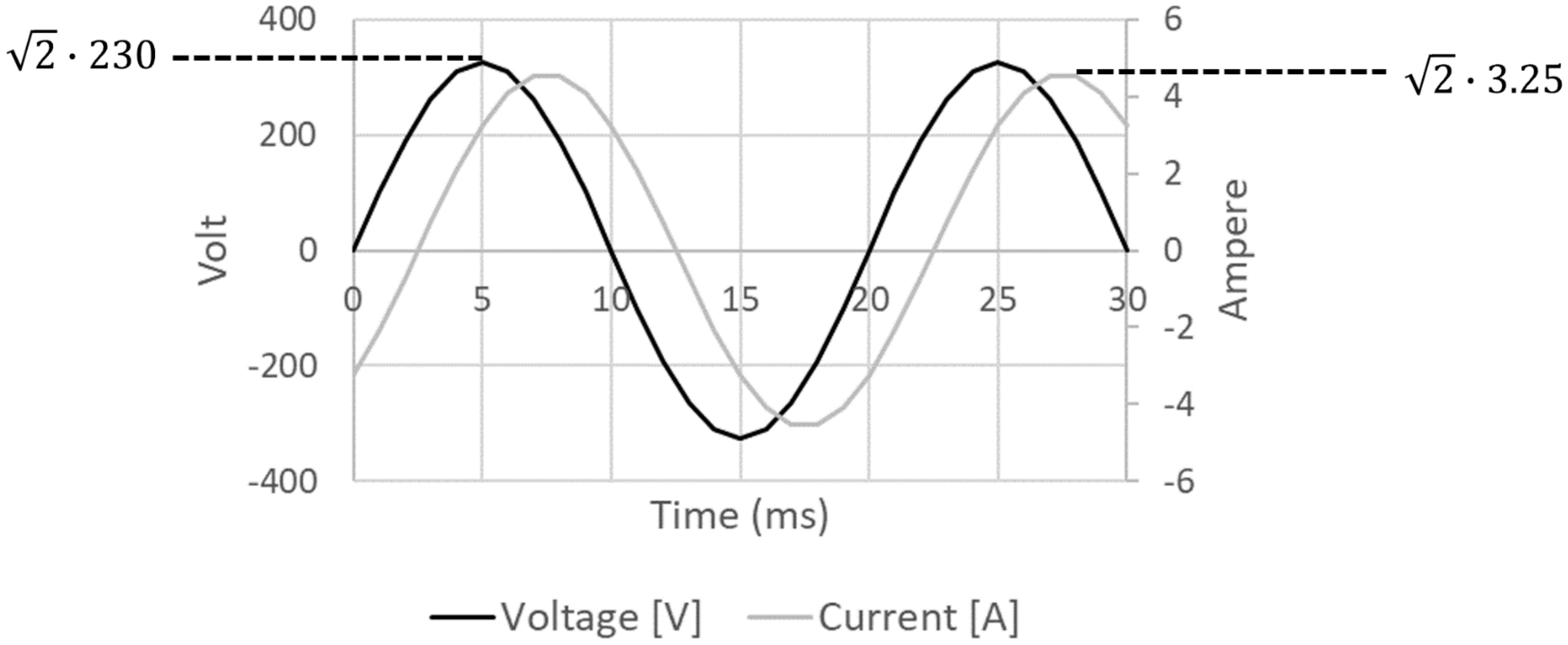
$$I = VY = 230 \cdot (0.01 - 0.01i) = 2.3 - 2.3i\ \mathrm{A}$$

The RMS of current is $\sqrt{2.3^2 + 2.3^2} = 3.25$ Ampere

The phase angle difference between current and voltage is $\arctan\left(\frac{-2.3}{2.3}\right) = -45°$

Current lags voltage by $\frac{45}{360}$ of a full cycle (where a full cycle lasts 20 ms), thus current peaks 2.5 ms after voltage

Example: current flowing through a passive element



Power flow

Consider a branch (m, n) of a circuit, V_{mn} the voltage applied along the branch, I_{mn} the current flowing through the branch

We can define (i) **apparent power** S_{mn}, (ii) **real power** P_{mn}, and (iii) **reactive power** Q_{mn} consumed on the line as:

$$S_{mn} = P_{mn} + Q_{mn}i = V_{mn}I_{mn}^*$$

Resistors, inductors, capacitors

Classification of passive electrical equipment based on admittance $Y = G + Bi$, where G is the **conductance**, and B is the **susceptance**

- Resistors: positive conductance ($G > 0$, $B = 0$), consume real power ($P_{mn} > 0$)
- Inductors: negative susceptance ($B < 0$, $G = 0$), consume reactive power ($Q_{mn} > 0$)
- Capacitors: positive susceptance ($G = 0$, $B > 0$), produce reactive power

Typically, transmission lines and transformers are reactive (δηλαδή $B < 0$) and slightly resistive (i.e. $G > 0$ but $G \ll |B|$)

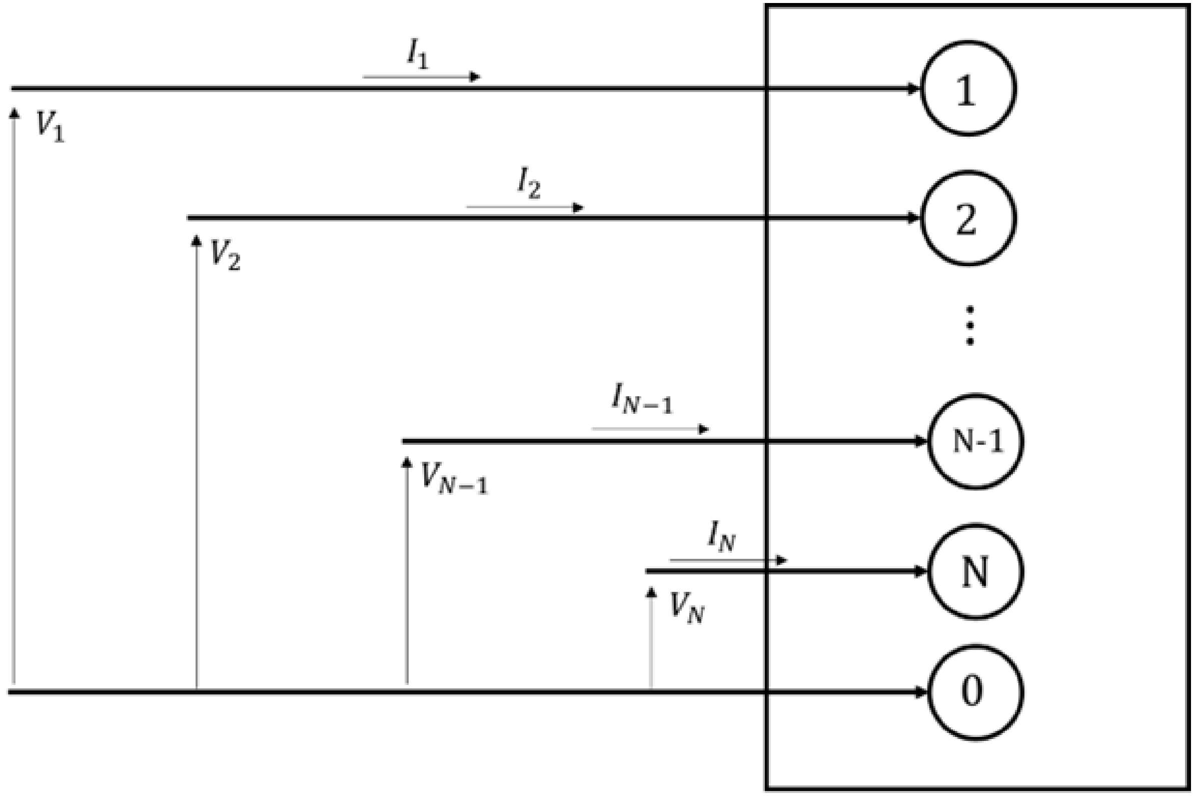
The power flow equations

Kirchhoff’s laws

Kirchhoff’s current law: the total current flowing into a node equals the total current flowing out of a node

Kirchhoff’s voltage law: accumulated voltage change across any loop of an electrical circuit equals zero

Model of a circuit with $N + 1$ buses



$$S_m = V_m I_m^* = P_m + Q_m i, m = 1, \dots, N$$

Performance equations in admittance form

Denote bus currents as $I_{\mathrm{bus}} = (I_1, I_2, \ldots, I_N)^T$, bus voltages as $V_{\mathrm{bus}} = (V_1, V_2, \ldots, V_N)^T$

Define the **admittance matrix** $Y_{\mathrm{bus}} \in \mathbb{C}^{N \times N}$ of the matrix:

- The non-diagonal element Y_{mn} is the negative of the admittance between bus m and bus n for $m \neq n$
- The diagonal element Y_{mm} is the sum of the admittance between node m and the ground plus the admittance between node m and all of its adjacent nodes

- **Performance equations** in admittance form:

$$I_{\mathrm{bus}} = Y_{\mathrm{bus}} V_{\mathrm{bus}}$$

Power flow equations

For bus m the following holds:

$$I_m = \sum_{n=1}^{N} Y_{mn} V_n$$

Conjugating this equation, we get

$$S_m = V_m I_m^* = V_m \sum_{n=1}^{N} Y_{mn}^* V_n^*$$

Separating into real and imaginary parts, we get the **power flow equations**

$$P_m = Re(V_m I_m^*) = Re\left(V_m \sum_{n=1}^{N} Y_{mn}^* V_n^*\right) \quad (B.1)$$

$$Q_m = Im(V_m I_m^*) = Im\left(V_m \sum_{n=1}^{N} Y_{mn}^* V_n^*\right) \quad (B.2)$$

Power flow equations in polar coorindates

In polar coordinates

$$P_m = |V_m| \sum_{n=1}^{N} |V_n| (G_{mn} \cos(\theta_{mn}) + B_{mn} \sin(\theta_{mn})) \quad (B.3)$$

$$Q_m = |V_m| \sum_{n=1}^{N} |V_n| (G_{mn} \sin(\theta_{mn}) - B_{mn} \cos(\theta_{mn})) \quad (B.4)$$

where $Y_{mn} = G_{mn} + B_{mn} i$ and θ_{mn} is the phase angle difference of voltages V_m and V_n

Slack bus, PQ buses, PV buses

There are N equations in complex numbers, thus $2N$ equations in real numbers, for a system with N nodes

There are $2N$ complex variables (apparent power and voltage at every node), which translate into $4N$ real variables

In a power flow problem, $2N$ of these variables are fixed, according to the following rule:

- There is a unique **swing bus** or **slack bus**, in which the voltage magnitude and voltage phase are fixed (with the phase commonly set equal to zero)
- The are M **load buses** or **P-Q buses**, for which the real and reactive power withdrawal are fixed
- There are $N - M - 1$ **production buses** or **P-V buses**, for which the real power and voltage magnitude are fixed

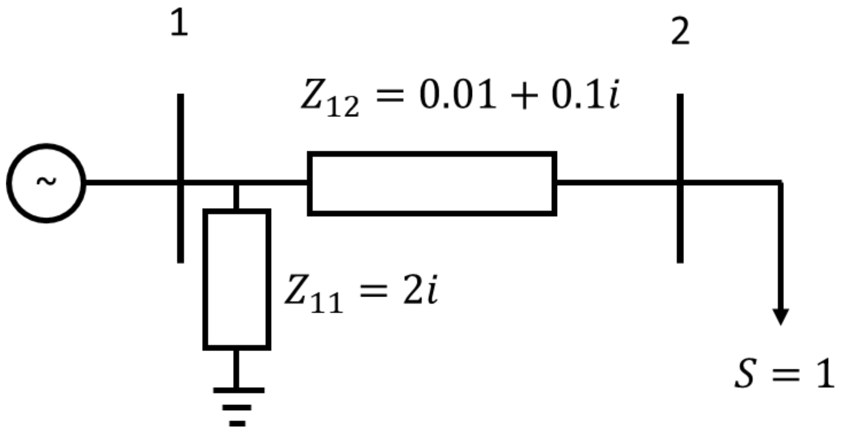
Tabular representation of the problem

	P	Q	$\|V\|$	θ	Πλήθος
Slack bus (generator)			√	√	1
PV buses (generators)	√		√		$N - M - 1$
PQ buses (loads)	√	√			M

- The non-trivial system has $N + M - 1$ variables (the red parts, $\theta_2, \dots, \theta_N, |V|_{n-m+1}, \dots, |V|_N$) in $N + M - 1$ equalities (power flow equations)
- The remaining (green) variables are solved for easily once we have computed the red ones (from the remaining equalities/power flow equations)

Two-node example

System buses:
- Bus 1: slack
- Bus 2: load/PQ



All values are in **per unit**

Admittance matrix:

$$Y_{\text{bus}} = \begin{bmatrix} \frac{1}{2i} + \frac{1}{0.01 + 0.1i} & -\frac{1}{0.01 + 0.1i} \\ -\frac{1}{0.01 + 0.1i} & \frac{1}{0.01 + 0.1i} \end{bmatrix}$$

$$= \begin{bmatrix} 0.99 - 10.4i & -0.99 + 9.9i \\ -0.99 + 9.9i & 0.99 - 9.9i \end{bmatrix}$$

Example: power flow equations

$$P_1 = G_{11} + |V_2|(G_{12}\cos(-\theta_2) + B_{12}\sin(-\theta_2))$$
$$= 0.99 + |V_2|(-0.99\cos(-\theta_2) + 9.9\sin(-\theta_2))$$

$$Q_1 = -B_{11} + |V_2|(G_{12}\sin(-\theta_2) - B_{12}\cos(-\theta_2))$$
$$= 10.4 + |V_2|(-0.99\sin(-\theta_2) - 9.9\cos(-\theta_2))$$

$$P_2 = |V_2|^2 G_{22} + |V_2|(G_{21}\cos(\theta_2) + B_{21}\sin(\theta_2)) \Rightarrow$$
$$-1 = 0.99|V_2|^2 + |V_2|(-0.99\cos(\theta_2) + 9.9\sin(\theta_2))$$

$$Q_2 = |V_2|(G_{21}\sin(\theta_2) - B_{21}\cos(\theta_2)) - |V_2|^2 B_{22} \Rightarrow$$
$$0 = |V_2|(-0.99\sin(\theta_2) - 9.9\cos(\theta_2)) + 9.9|V_2|^2$$

Example: solution

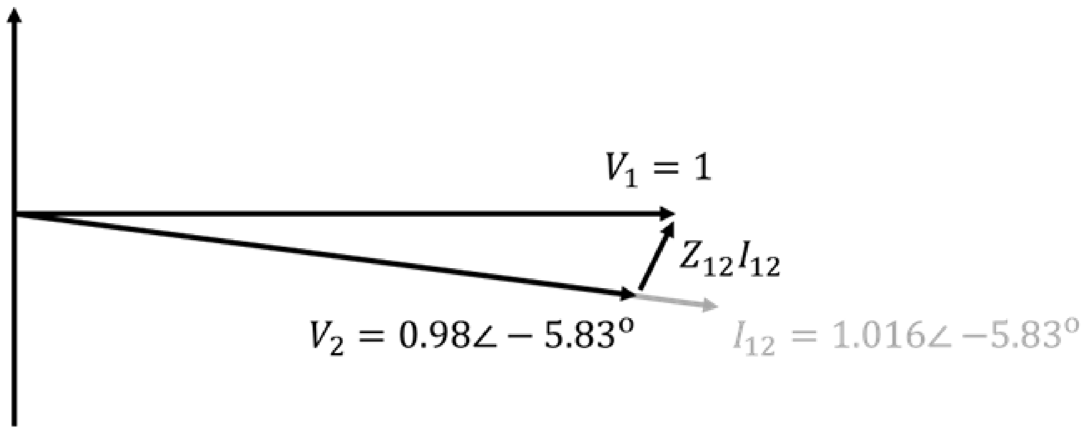
The solution of the power flow is:

$$P_1 = 1.010$$

$$Q_1 = 0.603$$

$$|V_2| = 0.985$$

$$\theta_2 = -5.83^o$$

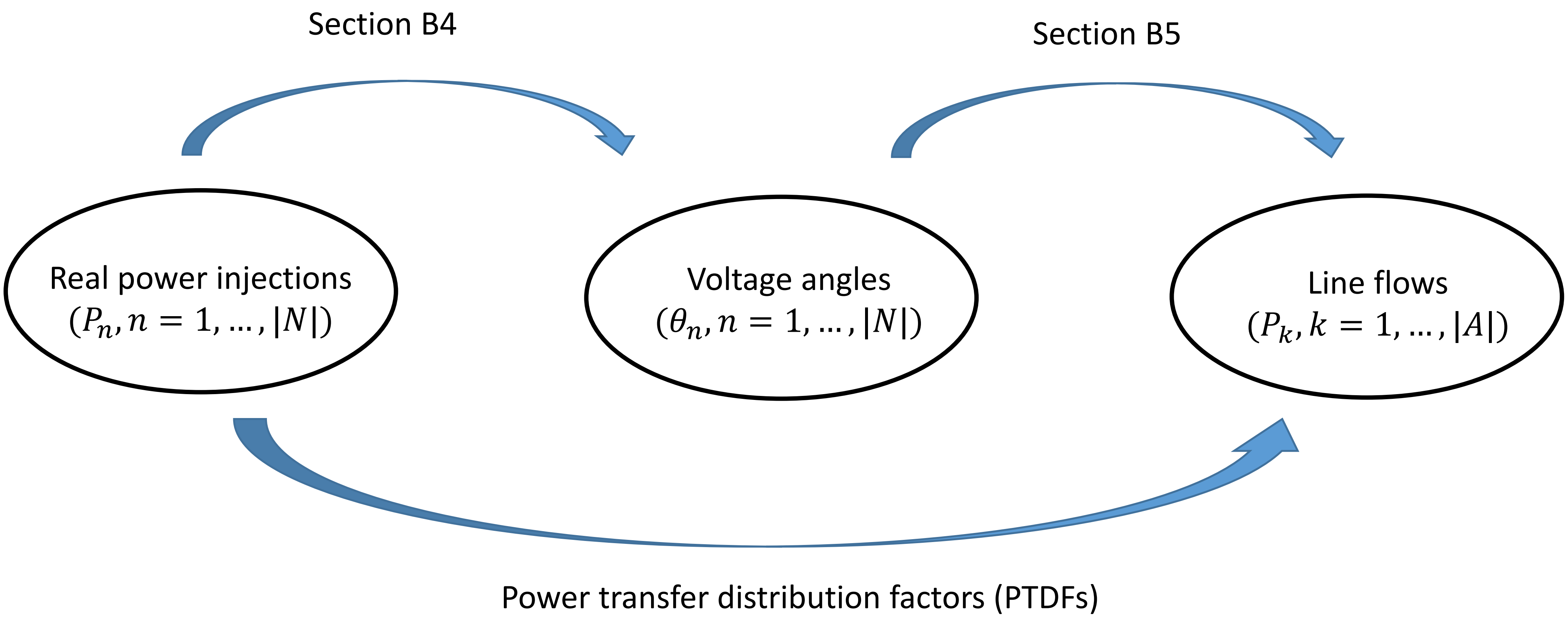


Losses are:

$$P_1 + P_2 = 1.010 - 1 = 0.010$$

From power injections to bus angles

Relations between injections, voltage angles and flows



From line flows to injections and from injections to line flows

A (non-radial) network has at least as many lines as nodes

Therefore, there are more flows than injections

If flows are given, we know what the injections are

The converse is typically not true: injections do not imply flows in a unique way

But they do imply them in the case of linearized power flow (we will see why now)

Linearized power flow equations

Consider the following approximations:

- Resistance is negligible: $G_{mn} = 0$
- Phase angle differences across branches $\theta_{mn} = \theta_m - \theta_n$ are small, so that $\sin(\theta_{mn}) \simeq \theta_{mn}$ and $\cos(\theta_{mn}) \simeq 1$
- Voltage magnitude on each bus is approximately equal to nominal: $|V_m| \simeq 1$

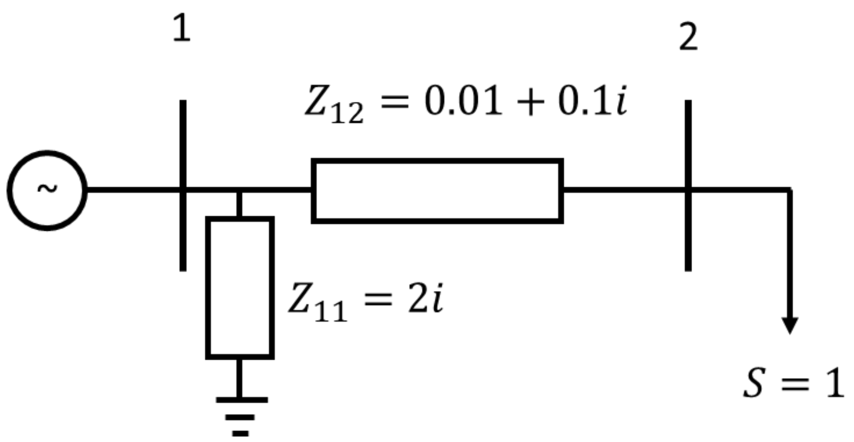
This leads to the **linearized power flow equations**

$$P_m = \sum_{n=1}^{N} B_{mn}(\theta_m - \theta_n) \quad (B.5)$$

Two-node example

Returning to the 2-node example of slide 28, we have:

$$P_1 = B_{12}(\theta_1 - \theta_2) = 9.9(0 - \theta_2)$$
$$P_2 = B_{21}(\theta_2 - \theta_1) \Rightarrow -1 = 9.9(\theta_2 - 0)$$



Solution:

$$P_1 = 1$$
$$\theta_2 = -5.79^{\text{o}}$$

Recall that the AC solution is $\theta_2 = -5.83^{\text{o}}$

Losses are $P_1 + P_2 = 0$

Further simplification of the linearization

- Since we ignore resistance on passive elements, let us assume that all conductances only have an imaginary part
- Thus $Y_{mn} = -y_{mn} = -\frac{1}{X_{mn}i} = iX_{mn}^{-1}$, where X_{mn} is the **susceptance** of line mn
- And since $Y_{mn} = B_{mn}i$, we conclude that $B_{mn} = X_{mn}^{-1}$
- Thus the linearized power flow equations are further simplified:

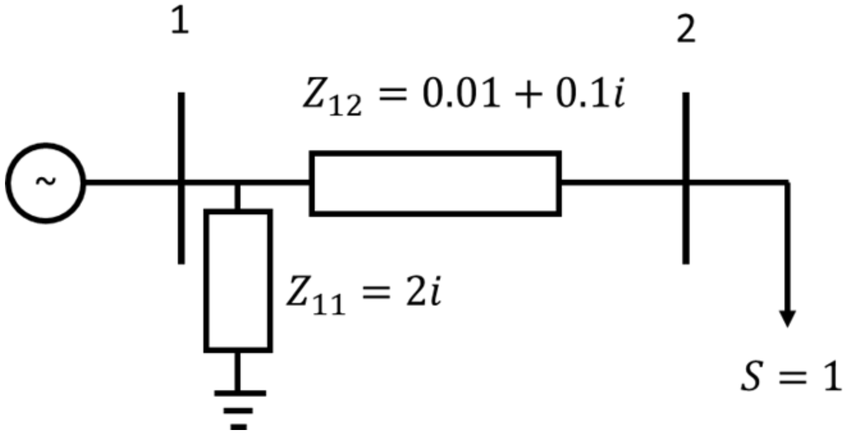
$$P_m = \sum_{n=1,n\neq m}^{N} \frac{1}{X_{mn}} \theta_m - \sum_{n=1,n\neq m}^{N} \frac{1}{X_{mn}} \theta_n \quad (B.6)$$

Two-node example

Returning to the two-node example:

$$P_1 = \frac{\theta_1 - \theta_2}{X_{12}} = \frac{0 - \theta_2}{0.1}$$

$$P_2 = \frac{\theta_2 - \theta_1}{X_{12}} \Rightarrow -1 = \frac{\theta_2 - 0}{0.1}$$



The solution is

$$P_1 = 1$$

$$\theta_2 = -5.73^{\text{o}}$$

Less accurate than the solution of slide 35 (-5.79^{o}), where the exact AC solution is -5.83^{o}, but still quite accurate

Losses: $P_1 + P_2 = 0$

The matrix T^F

The injections of real power P_m are linearly dependent on voltage angles, θ_m

This dependence is described by the weighted Laplacian of the network graph, where the weights of lines are given by the parameters B_{mn}

Define $T^F = (T_{mn}), m, n \in \{1, \ldots, N\}$, where T^F is a symmetric $N \times N$ matrix

$$T_{mn}^F = \begin{cases} -\dfrac{1}{X_{mn}}, (m,n) \in A, m \neq n \\ \displaystyle\sum_{n'=1, n' \neq m}^{N} \dfrac{1}{X_{mn}}, m = n \\ 0, (m,n) \notin A \end{cases} \qquad (B.7)$$

Relation between injections and voltage angles

The linear mapping from voltage angles $\theta^F \in \mathbb{R}^N$ to injections $P^F \in \mathbb{R}^N$ at each bus is described as:

$$P^F = T^F \theta^F \quad (B.8)$$

The proposition of slide 9 guarantees that, if the graph is connected, then the degree of the Laplacian T^F is $N-1$

Thus there are $N-1$ constraints that contain the full information of the linear mapping, and therefore one constraint can be removed from the linear system

Any choice of $N-1$ constraints keeps the full information of the mapping, because any line equals minus the sum of the rest of the lines (conservation of energy):

$$\sum_{m=1}^{N} P_m = 0 \ \ (B.9)$$

Hub node

Given that $T^F = (T^F)^T$, this means that column i is minus the sum of the other columns of the matrix

We can therefore select a sub-matrix of T^F of dimension $(N-1) \times (N-1)$, which we denote as T, by removing row and column i of T^F

Since the degree of the remaining matrix is $N-1$, the matrix T is invertible

The node corresponding to the column/row that is removed is the **hub node**, which is the analog of the slack bus in power flow:

- Its voltage angle is zero, $\theta_h = 0$
- The injection of real power in this bus is:

$$P_h = -\sum_{n \in \{1,..,N\} - \{h\}} P_n$$

One-to-one relation between net injections and voltage angles

The power flow equations can be expressed equivalently by ignoring the voltage angle of the hub node (set equal to zero) and the injection of real power at the hub node (since it is implied from the injections of all other buses)

This leads to the following $(N-1) \times (N-1)$ linear system:

$$P = T\theta, \qquad (B.10)$$

where $P = (P_m), m \in \{1, \dots, N\} - \{h\}$ and $\theta = (\theta_m), m \in \{1, \dots, N\} - \{h\}$

From voltage angles to line flows

Line flows as a function of angles

Proposition: The power flow across line (m, n) is

$$P_{mn} = \frac{1}{X_{mn}}(\theta_m - \theta_n) \quad (B.11)$$

Power transfer distribution factors

Given a line k and a bus n, the **power transfer distribution factor (PTDF)** is the amount of power flow induced on line k by a tranfer of 1 MW of power from bus n to the hub node

The value of a PTDF depends on the choice of hub node

Computing PTDFs

Define the matrix M as $M = (M_{kn}), k \in A, n \in \{1, \ldots, N\} - \{h\}$, where

$$M_{kn} = \begin{cases} \frac{1}{X_k}, & \text{if } k = (n,\cdot), n \neq h \\ -\frac{1}{X_k}, & \text{if } k = (\cdot, n), n \neq h \\ 0, & \text{otherwise} \end{cases} \quad (B.12)$$

By the definition of M, and from equation (B.11) we conclude that

$$P_L = M\theta \quad (B.13)$$

where P_L is the vector of power flows along the lines of the network

Computing PTDFs

From equation (B.10)

$$P_L = MT^{-1}P \quad (B.14)$$

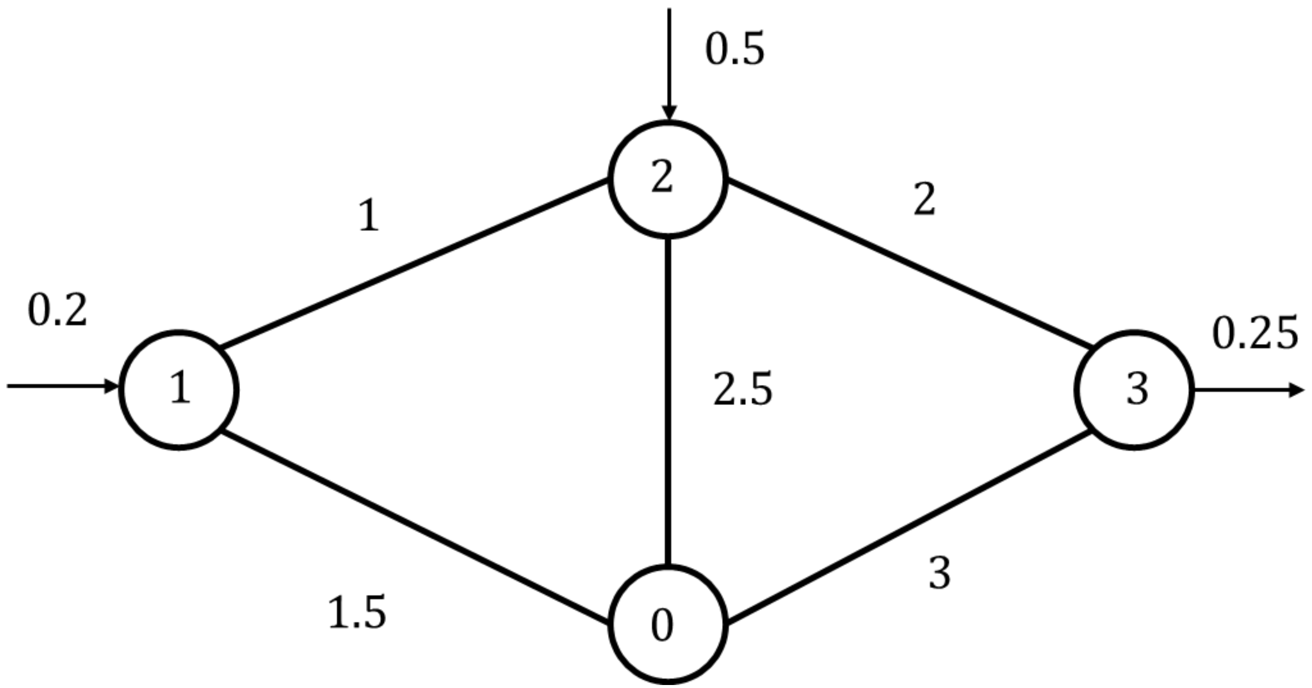
This is the desired mapping from injections P to line flows P_L

The PTDF of bus n on line k (denoted as F_{kn}) is:

$$F_{kn} = M_k^T (T^{-1})_n,$$

where M_k^T is the k-th row of M, and $(T^{-1})_n$ is the n-th column of T^{-1}

Example: 4-bus network



The numbers on lines correspond to line inductances X_{mn}

The numbers next to arrows correspond to real power injections

Example: 4-bus network

Compute the PTDF of node 1 on line 2-3, where node 0 is the hub node

Matrix T is:

$$T = \begin{bmatrix} \frac{1}{1} + \frac{1}{1.5} & -\frac{1}{1} & 0 \\ -\frac{1}{1} & \frac{1}{1} + \frac{1}{2.5} + \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2} & \frac{1}{2} + \frac{1}{3} \end{bmatrix}$$

Inverting T:

$$T^{-1} = \begin{bmatrix} 0.96 & 0.6 & 0.36 \\ 0.6 & 1.0 & 0.6 \\ 0.36 & 0.6 & 1.56 \end{bmatrix}$$

For power injection $P = (0.2, 0.5, -0.25)^T$, bus angles are

$$\theta = T^{-1}P = \begin{bmatrix} 0.96 & 0.6 & 0.36 \\ 0.6 & 1.0 & 0.6 \\ 0.36 & 0.6 & 1.56 \end{bmatrix} \begin{bmatrix} 0.2 \\ 0.5 \\ -0.25 \end{bmatrix} = \begin{bmatrix} 0.402 \\ 0.470 \\ -0.018 \end{bmatrix}$$

The power flow on each line is

$$P_{12} = \frac{\theta_1 - \theta_2}{X_{12}} = \frac{0.402 - 0.47}{1} = -0.068$$

$$P_{10} = \frac{\theta_1 - \theta_0}{X_{10}} = \frac{0.402 - 0}{1.5} = 0.268$$

$$P_{23} = \frac{\theta_2 - \theta_3}{X_{23}} = \frac{0.47 - (0.018)}{2} = 0.244$$

$$P_{20} = \frac{\theta_2 - \theta_0}{X_{20}} = \frac{0.47 - 0}{2.5} = 0.188$$

$$P_{30} = \frac{\theta_3 - \theta_0}{X_{30}} = \frac{-0.018 - 0}{3} = -0.006$$

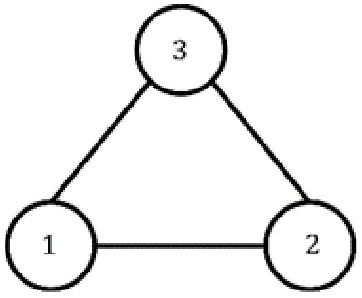
Representing $M_k^T \in \mathbb{R}^3$ as the k-th row of matrix M, the matrix M that determines line flows as a function of bus angles, $P_L = M\theta$, is

$$M = \begin{bmatrix} M_{01}^T \\ M_{12}^T \\ M_{23}^T \\ M_{02}^T \\ M_{03}^T \end{bmatrix} = \begin{bmatrix} -\frac{1}{1.5} & 0 & 0 \\ \frac{1}{1} & -\frac{1}{1} & 0 \\ 0 & \frac{1}{2} & -\frac{1}{2} \\ 0 & -\frac{1}{2.5} & 0 \\ 0 & 0 & -\frac{1}{3} \end{bmatrix}$$

The change in flow on line (2, 3) caused by the injection of one MW of power in node 1 is

$$F_{23,1} = M_{23}^T (T^{-1})_1 = (0, 0.5, -0.5)(0.96, 0.6, 0.36)^T = 0.12$$

Example: symmetric 3-node network



Denote X as the reactance of each line

Compute the PTDF matrix of the network when node 3 is the hub node

Example: symmetric 3-node network

Matrix T:

$$T = \begin{bmatrix} \frac{1}{X} + \frac{1}{X} & -\frac{1}{X} \\ -\frac{1}{X} & \frac{1}{X} + \frac{1}{X} \end{bmatrix} = \frac{1}{X} \begin{bmatrix} 2 & -1 \\ -1 & 2 \end{bmatrix}$$

Inverting, we compute matrix T^{-1}:

$$T^{-1} = X \begin{bmatrix} 0.667 & 0.333 \\ 0.333 & 0.667 \end{bmatrix}$$

Matrix M:

$$M = \begin{bmatrix} M_{12}^T \\ M_{23}^T \\ M_{13}^T \end{bmatrix} = \begin{bmatrix} \frac{1}{X} & -\frac{1}{X} \\ 0 & \frac{1}{X} \\ \frac{1}{X} & 0 \end{bmatrix}$$

PTDF computation:

$$F_{12,1} = \left(\frac{1}{X}, -\frac{1}{X}\right)(0.667X, 0.333X)^T = 0.333$$

$$F_{13,1} = \left(\frac{1}{X}, 0\right)(0.667X, 0.333X)^T = 0.667$$

$$F_{23,1} = \left(0, \frac{1}{X}\right)(0.667X, 0.333X)^T = 0.333$$

$$F_{12,2} = \left(\frac{1}{X}, -\frac{1}{X}\right)(0.333X, 0.667X)^T = -0.333$$

$$F_{13,2} = \left(\frac{1}{X}, 0\right)(0.333X, 0.667X)^T = 0.333$$

$$F_{23,2} = \left(0, \frac{1}{X}\right)(0.333X, 0.667X)^T = 0.667$$

Physical intuition: current splits in a way that is inversely proportional to reactance

Losses

Losses on a line

- Complex power losses along a line:

$$S_k = V_m I_{mn}^* + V_n I_{nm}^* = V_m I_{mn}^* - V_n I_{mn}^* = (V_m - V_n) I_{mn}^*$$
$$= (V_m - V_n) y_{mn}^* (V_m - V_n)^* = |V_m - V_n|^2 y_{mn}^*$$

- After algebraic manipulations:

$$|V_m - V_n|^2 = |V_m|^2 + |V_n|^2 - 2|V_m||V_n| \cos(\theta_m - \theta_n)$$

- Isolating the real part of S_k :

$$L_k = g_{mn}\left(|V_m|^2 + |V_n|^2 - 2|V_m||V_n| \cos(\theta_m - \theta_n)\right)$$

Approximating losses

- Taylor expansion of $\cos(\theta_m - \theta_n) \simeq 1 - \frac{(\theta_m - \theta_n)^2}{2}$:

$$L_k \simeq g_{mn}\left(|V_m|^2 + |V_n|^2 - 2|V_m||V_n| + |V_m||V_n|(\theta_m - \theta_n)^2\right)$$

- And since $|V_m| \simeq |V_n| \simeq 1$ in linearized power flow:

$$L_k \simeq g_{mn}(\theta_m - \theta_n)^2$$

- Further assumptions of linearized power flow imply:

$$g_{mn} = \frac{R_{mn}}{R_{mn}^2 + X_{mn}^2} \simeq \frac{R_{mn}}{X_{mn}^2}$$

$$(\theta_m - \theta_n)^2 = P_{mn}^2 X_{mn}^2$$

- Finally arriving to

$$L_k \simeq R_{mn} P_{mn}^2$$

Example: approximating losses

- Returning to the two-node example of slide 28:

$$L_{12} = R_{12} P_{12}^2 = 0.01 \cdot 1^2 = 0.01$$

- Here, we use the line flow solution P_{12} of the linearized model (slide 37)

Losses as a function of power injections

- First-order Taylor approximation of square of losses on line k:

$$P_k^2 \simeq \bar{P}_k^2 + 2\bar{P}_k\left(P_k - \bar{P}_k\right) = -\bar{P}_k^2 + 2 \cdot \bar{P}_k \cdot P_k$$

- Here, $\bar{P}_k$ is the base dispatch point of the system
- Since $P_k = \sum_{n \in N} PTDF_{kn} \cdot P_n$, we finally have

$$L = \sum_{k \in K} L_k = \sum_{k \in K} R_k \cdot (-\bar{P}_k^2 + 2 \cdot \bar{P}_k \cdot \sum_{n \in N} PTDF_{kn} \cdot P_n)$$

References

[1] A. Papavasiliou, Optimization Models in Electricity Markets, Cambridge University Press

https://www.cambridge.org/highereducation/books/optimization-models-in-electricity-markets/0D2D36891FB5EB6AAC3A4EFC78A8F1D3#overview