Risk Management

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Source: chapter 9, Papavasiliou [1]

Outline

- Forward contracts
  - The virtues of forward contracts
  - The price of forward contracts
  - Contracts for differences and power purchase agreements
- Financial transmission rights
  - FTR auctions
  - The virtues of FTRs
- Callable forward contracts and reliability options
  - The price of callable forward contracts
  - The virtues of callable forward contracts
- Modeling risk aversion
  - Value at risk and conditional value at risk
  - Worst-case characterization of coherent risk measures
  - Back-propagation
  - Other ways of representing risk aversion

Forward contracts

The virtues of forward contracts

The price of forward contracts

Contracts for differences and power purchase agreements

Forward contracts

Forward contracts: financial instruments for trading a commodity in a price fixed in advance

Characterized by:

- Selling price f_t
- Quantity x of traded commodity
- Delivery time T of commodity/**expiration date** of forward contract

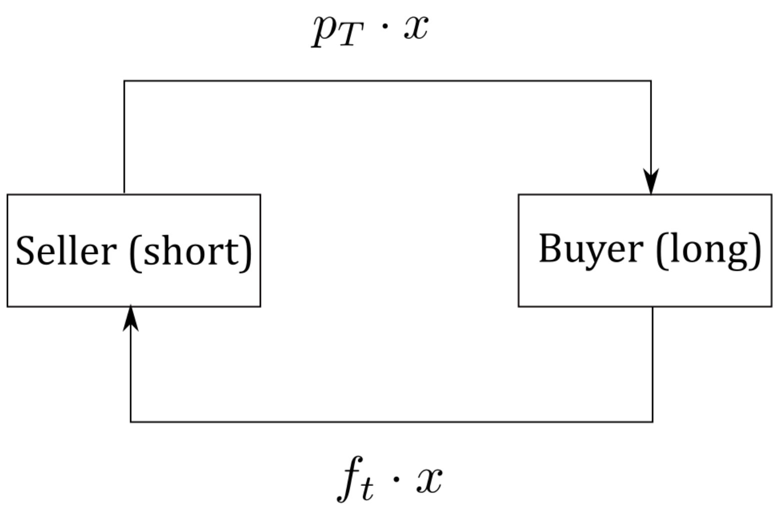
Definition

Seller: Seller of a forward contract with expiration date T sells contract at $t < T$ for a price f_t. Seller has a **short** position.

Buyer: Buyer of a forward contract with expiration date T buys contract at $t < T$ for a price f_t. Buyer has a **long** position.

Obligations and payoffs: At time $t < T$, buyer pays seller $f_t \cdot x$. At time $t = T$, seller pays buyer $p_T \cdot x$. The price p_T is the real-time price of the underlying commodity.

Payments



Forward contracts

The virtues of forward contracts

The price of forward contracts

Contracts for differences and power purchase agreements

Virtues of forward contracts

- Hedging
- Forward contracts do not distort real-time incentives
- Forward contracts can be traded

Trading at fixed prices through forward contracts

- Producers: sell forward, produce in real time
  - $+f_t \cdot x$ (from selling forward contract)
  - $+p_T \cdot x$ (from producing in real-time market)
  - $-p_T \cdot x$ (from settling forward contract)

- Consumers: buy forward, consume in real time
  - $-f_t \cdot x$ (from buying forward contract)
  - $-p_T \cdot x$ (from consuming in real-time market)
  - $+p_T \cdot x$ (from settling forward contract)

Hedging risk without distorting real-time incentives

Suppose producer buys forward contact for x units at price f_t and produces q in real time

Producer is paid:

$$R = f_t \cdot x + p_T \cdot (q - x)$$

where p_T is real-time price

- At T, producer only influences $p_T \cdot q \Rightarrow$ correct incentives, because the real-time price p_T is applied to the real-time decision q
- By producing $q = x$, producer receives price $f_t \Rightarrow$ hedging

Futures contracts

Futures contracts: standardized forward contracts with rigid terms that are exchanged in a clearing house

- Default risk is reduced, carried by clearing house (+)
- Enhanced liquidity (+)
- No concerns of credit-worthiness for traders (+)
- Less flexibility (-)

Integration with power system operations

- Forward contracts
  - Suppliers and consumers can enter a forward contract *in advance*
  - In *real time*
    - Suppliers submit bid at price floor
    - Consumes submit demand bid at price ceiling

- Futures contracts can be traded with the system operator
  - Sellers of futures pay system operator
  - Buyers of futures get paid by system operator
  - System operator gets information about supply-demand balance from the contracts

Forward contracts

The virtues of forward contracts

The price of forward contracts

Contracts for differences and power purchase agreements

Price of a forward contract

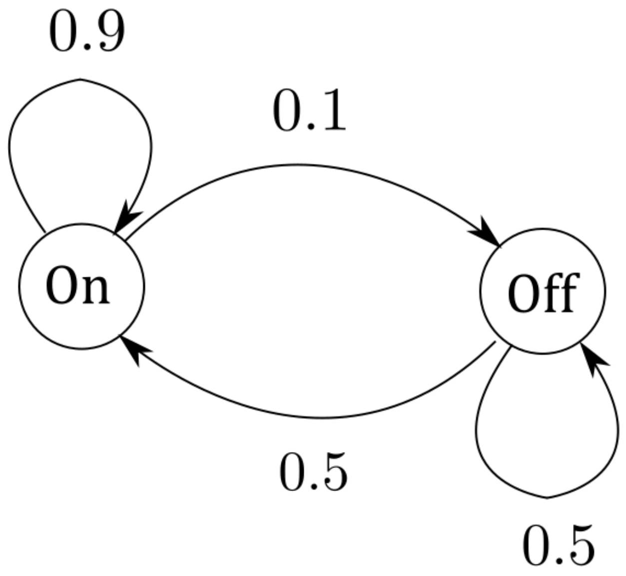
- Given risk neutral market agents with same beliefs about the distribution of future real-time price p_T:

$$f_t = \mathbb{E}[p_T | \xi_{[t]}]$$

$\xi_{[t]}$: available information at time t

Example

- Linear demand function:
$$D(p) = 1620 - 4p$$
- Generator 1
  - Capacity: 295 MW
  - Marginal cost: 65.1 $/MWh
- Generator 2
  - Capacity: 1880 MW
  - Marginal cost: 11.8 $/MWh
  - Failures described by Markov chain



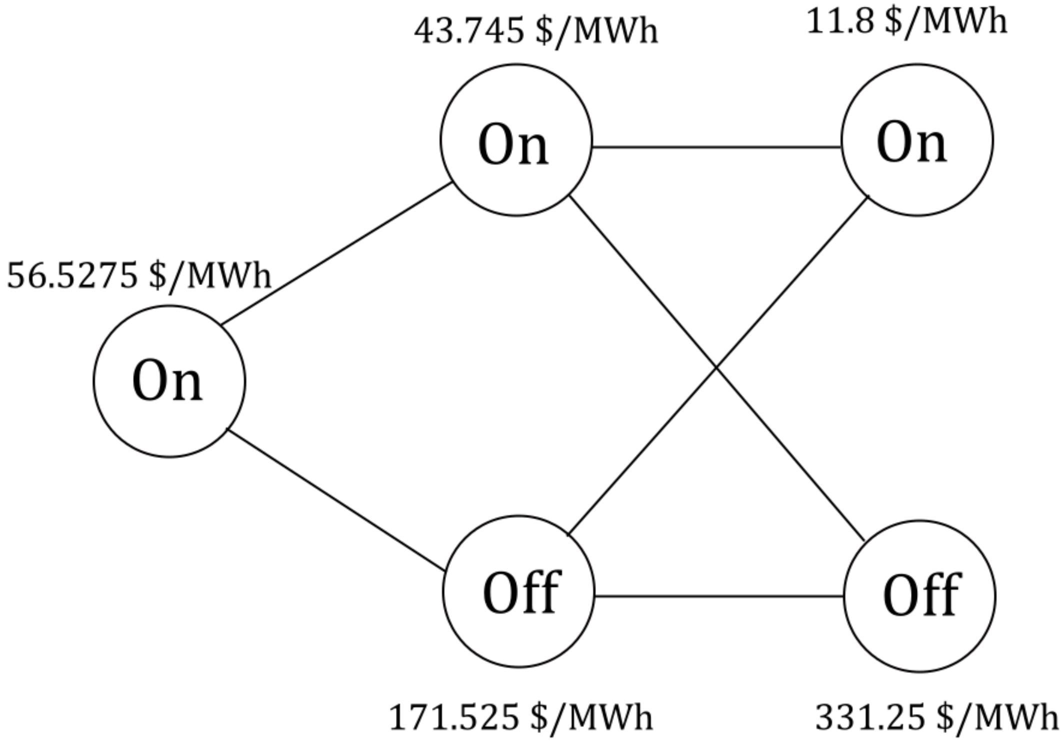
Computing forward prices

- Period 2 (you should compute this)
  - Generator 2 off: 295 MW at 331.25 $/MWh
  - Generator 2 on: 1572.8 MW at 11.8 $/MWh
- Period 1

$$f_1 = \begin{cases} 0.9 \cdot 11.8 + 0.1 \cdot 331.25 = 43.745 \dfrac{\$}{\text{MWh}}, & \xi_{[1]} = \text{On} \\ 0.5 \cdot 11.8 + 0.5 \cdot 331.25 = 171.525 \dfrac{\$}{\text{MWh}}, & \xi_{[1]} = \text{Off} \end{cases}$$

- Period 0 (assuming generator 2 is on)

$$f_0 = 0.9 \cdot 43.745 + 0.1 \cdot 171.525 = 56.5275 \frac{\$}{\text{MWh}}$$


43.745 $/MWh
11.8 $/MWh
56.5275 $/MWh
On
On
On
Off
Off
171.525 $/MWh
331.25 $/MWh

Forward contracts

The virtues of forward contracts

The price of forward contracts

Contracts for differences and power purchase agreements

Contracts for differences

- **Contracts for differences (CfDs)**: Alternative derivatives that serve same function as forward contract

Seller: A seller sells a CfD with expiration date T at time $t < T$ for x units of a commodity

Buyer: A buyer buys a CfD with expiration date T at time $t < \mathrm{T}$ for x units of a commodity

Obligations and payoffs: At time T the buyer pays the seller $(f_t - p_T) \cdot x$, where p_T is the price of the commodity at time T

Trading at fixed prices though CfDs

Buyer of CfD (consumer) consumes x at T:

- Pays $(f_t - p_T) \cdot x$ for CfD
- Pays $p_T \cdot x$ to spot market

Seller of CfD (supplier) produces x at T:

- Paid $(f_t - p_T) \cdot x$ for CfD
- Paid $p_T \cdot x$ from spot market

Power purchase agreements

Power purchase agreements (PPAs): bilateral agreements for trading electricity at a fixed price

In practice PPAs are often used for financing renewable energy projects:

- Sellers of PPAs: owners of existing or potential renewable projects
- Buyers of PPAs: large electricity consumers, such as corporations with stewardship goals in the shift towards consuming renewable energy

Financial transmission rights

FTR auctions

The virtues of FTRs

The need for financial transmission rights

Forward contracts are adequate for trading electricity at a fixed price in a market without congestion

What happens if there is congestion?

Example

Generator A wants to trade 400 MW with consumer B at 40 $/MWh

Generator sells forward contract for 400 MW at 40 $/MWh στο φορτίο

Suppose $p_A = p_B = 50$ $/MWh

Cash flows to producer:

- $+40 \cdot 400 = +\$16000$ (sell forward)
- $+50 \cdot 400 = +\$20000$ (produce in real-time market)
- $-50 \cdot 400 = -\$20000$ (settle forward)

Cash flows to load: $-40 \cdot 400 - 50 \cdot 400 + 50 \cdot 400 = -\$16000$

Result: parties trade at 40 $/MWh

Suppose $p_A = 36$ \$/MWh, $p_B = 45$ \$/MWh

Suppose generator sells forward contract for 400 MW in location A

Cash flows to producer: $+40 \cdot 400 + 36 \cdot 400 - 36 \cdot 400 = +\$16000$

Cash flows to load: $-40 \cdot 400 - 45 \cdot 400 + 36 \cdot 400 = -\$19600$

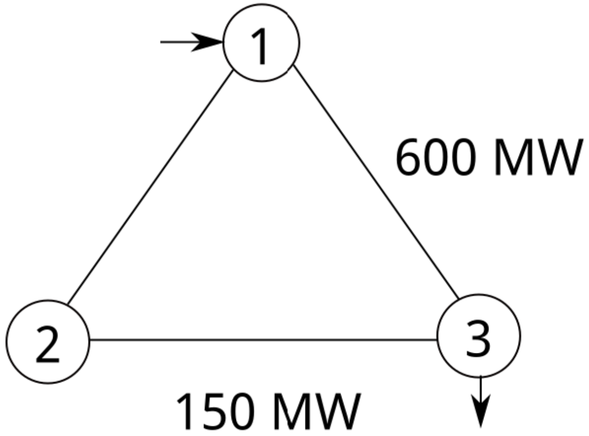
Result: generator paid 40 \$/MWh, load pays 49 \$/MWh ⇒ load pays transportation cost $p_B - p_{\mathrm{A}} = 9$ €/MWh

Transmission rights

In order to develop financial instruments that hedge against locational price differences it is necessary to define *rights* for the usage of lines

- **Contract paths**: right to ship power over paths
  - Ignores Kirchhoff's laws
  - Failed
- **Financial transmission rights (FTRs)** (Hogan, 1992 [2]): rights to ship power *between nodes*

Failure of contract paths (Hogan, 1992)



- Line 1-3 limit: 600 MW
- Line 2-3 limit: 150 MW
- Lines have identical characteristics

Why contact paths fail

Suppose that we wish to define transmission rights from producers to consumers

How many rights for path 1-3?

- Option 1: 300 MW (capacity of line 1-3)
  - Could overload line 2-3
  - Disadvantage: inefficient (suppose that cheaper generators are in node 2)
- Option 2: 300 MW (to avoid overloading line 1-3)
  - But if there are loads in node 2 trading with generators in node 3 then line 1-3 can handle more than 300 MW of trade on the path 1-3

Conclusion: the network capacity that is traded in contract paths is not given, but depends on the state of the system

Financial transmission rights

Seller: At time $t < T$ the seller sells a financial transmission right for shipping x units of power from location A to location B with expiration date T

Buyer: At time $t < T$ the buyer of an FTR with expiration date T buys the contract

- *Obligations and payoffs*: At time T the seller pays the buyer of the FTR $(p_B - p_A) \cdot x$ (p_A, p_B are the LMPs)

Example revisited

Load B buys forward contract from generator A and FTR from A to B

Cash flows to load:

- $-40 \cdot 400 = -\$16000$ (buying forward)
- $-45 \cdot 400 = -\$18000$ (consuming in real-time market)
- $+36 \cdot 400 = +\$14400$ (settling forward)
- $+9 \cdot 400 = +\$3600$ (settling FTR)

Result: load pays 40 $/MWh

Bilateral trade at fixed prices

Producer sells forward contract to load and load buys FTR from generator location (A) to load location (B)

Cash flows to producer:

- $+f_t \cdot x$ (selling forward)
- $+p_A \cdot x$ (producing in real-time market)
- $-p_A \cdot x$ (settling forward)

Cash flows to consumer:

- $-f_t \cdot x$ (buying forward)
- $-p_B \cdot x$ (consuming in real-time market)
- $+p_A \cdot x$ (settling forward)
- $+(p_B - p_A) \cdot x$ (settling FTR)

Result: trade in fixed price f_t which is known in advance

Financial transmission rights

FTR auctions

The virtues of FTRs

FTR auctions

Default seller of FTRs: system operators

Simultaneous feasibility of FTRs: Allocation of FTRs must respect transmission constraints

Recall **congestion rent**: LMP auction payments

Revenue adequacy: LMP auction payments are enough to cover FTR payments if FTRs are simultaneously feasible

Proof: we first recall that congestion rent is non-negative, then show it exceeds FTR payments

Recall OPF

(DCOPF):
$$\max_{p,d,f,r} \sum_{l\in L} \int_0^{d_l} MB_l(x)\, dx - \sum_{g\in G} \int_0^{p_g} MC_g(x) dx$$

(λ_k^+):
$$f_k \leq T_k, k \in K$$

(λ_k^-):
$$-f_k \leq T_k, k \in K$$

(ψ_k):
$$f_k - \sum_{n\in N} F_{kn} \cdot r_n = 0, k \in K$$

(ρ_n):
$$r_n - \sum_{g\in G_n} p_g + \sum_{l\in L_n} d_l = 0, n \in N$$

(φ):
$$\sum_{n\in N} r_n = 0$$

$$p_g \geq 0, g \in G$$

$$d_l \geq 0, l \in L$$

Congestion rent is non-negative

Congestion rent is non-negative, and given by the following expression:

$$CR = \sum_{n \in N} \rho_n \cdot (\sum_{l \in L_n} d_l - \sum_{g \in G_n} p_g) = \sum_{k \in K} (\lambda_k^+ + \lambda_k^-) \cdot T_k$$

Proof: If identity is true, then since $\lambda_k^+ \geq 0, \lambda_k^- \geq 0$ congestion rent is non-negative

$$\sum_{n\in N} \rho_n \cdot (\sum_{l\in L_n} d_l - \sum_{g\in G_n} p_g) =$$

By definition of r_n

$$-\sum_{n\in N} \rho_n \cdot r_n =$$

Since $\rho_n = \sum_{k\in K} F_{kn} \cdot \psi_k - \varphi$ and $\psi_k = \lambda_k^- - \lambda_k^+$ and $\sum_{n\in N} r_n = 0$

$$\sum_{k\in K} (\lambda_k^+ - \lambda_k^-) \cdot \sum_{n\in N} F_{kn} \cdot r_n =$$

By definition of f_k

$$\sum_{k\in K} (\lambda_k^+ - \lambda_k^-) \cdot f_k =$$

From $0 \leq \lambda_k^+ \perp T_k - f_k \geq 0$ and $0 \leq \lambda_k^- \perp T_k + f_k \geq 0$

$$\sum_{k\in K} (\lambda_k^+ + \lambda_k^-) \cdot T_k$$

Congestion rent and FTR payments

FTRs pay to their holders

$$-\sum_{n\in N}\rho_n\cdot\tilde{r}_n$$

where $\tilde{r}_n$ is a feasible (not necessarily optimal) dispatch

Congestion rent is adequate to cover FTR payments:

$$-\sum_{n\in N}\rho_n\cdot r_n\geq-\sum_{n\in N}\rho_n\cdot\tilde{r}_n$$

Proof: from slide 36,

$$-\sum_{n \in N} \rho_n \cdot (r_n - \tilde{r}_n) = \sum_{k \in K} (\lambda_k^+ - \lambda_k^-) \cdot \left(f_k - \tilde{f}_k\right)$$

where:

- λ_k^+, λ_k^- are dual optimal multipliers
- f_k are flows corresponding to r_n
- $\tilde{f}_k$ are flows corresponding to $\tilde{r}_n$

Consider three cases:

- $f_k = T_k$ (which implies $\lambda_k^- = 0$)
- $f_k = -T_k$ (which implies $\lambda_k^+ = 0$)
- $-T_k < f_k < T_k$ (which implies $\lambda_k^+ = \lambda_k^- = 0$)

Financial transmission rights

FTR auctions

The virtues of FTRs

Physical transmission rights

Physical transmission rights (PTRs): provide *exclusive* access to the holder of the rights, no financial payoff

FTRs are purely financial, do not interfere with efficient dispatch $\neq$ PTRs can lead to inefficiency

Callable forward contracts

The price of callable forward contracts

The virtues of callable forward contracts

Call options

Seller: Seller of a call option with expiration date T and **strike price** k sells option at $t < T$ for amount x of underlying commodity

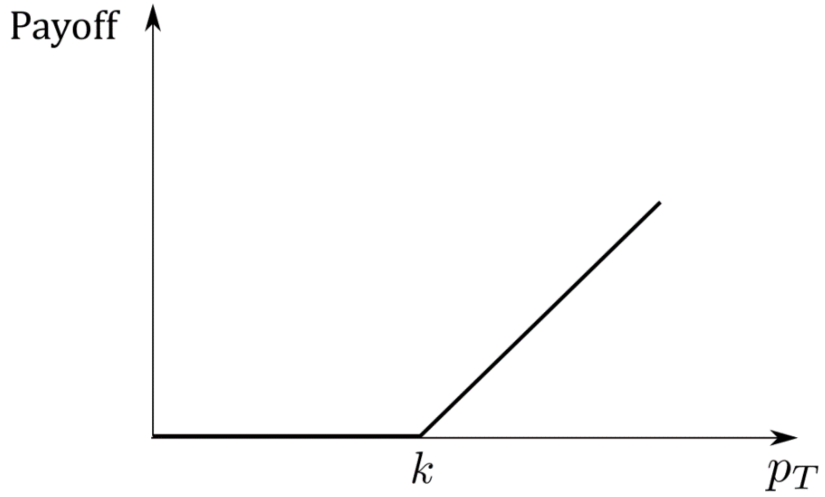
Buyer: Buyer of call option with expiration date T and strike price k buys contract at $t < T$ for amount x of underlying commodity

- *Obligations and payoffs*: At $t < T$ buyer pays seller the price of the call option. At T seller pays buyer $\max(p_T - k, 0) \cdot x$, where p_T is spot price of the underlying commodity.

The function of call options

The buyer of the option has the right, but not the obligation, to buy the commodity at strike price k at expiration

- $p_T \leq k$: no value from call option
- $p_T > k$: buyer receives $p_T - k$, can buy the commodity in the spot market with net expense of k



Reliability options

- Call options can be used as instruments for hedging the risk for buyers who do not want to be exposed to high real-time prices of commodities, as well as investors who build generation capacity
- Call options can specifically be bundled with capacity markets in order to allow generators to trade the payoff of the market during periods of stress with a forward payment
- Call options that serve this purpose are referred to as **reliability options**

Callable forward

Seller: Seller of a callable forward with expiration date T and strike price k sells contract at $t < T$ for amount x of underlying commodity

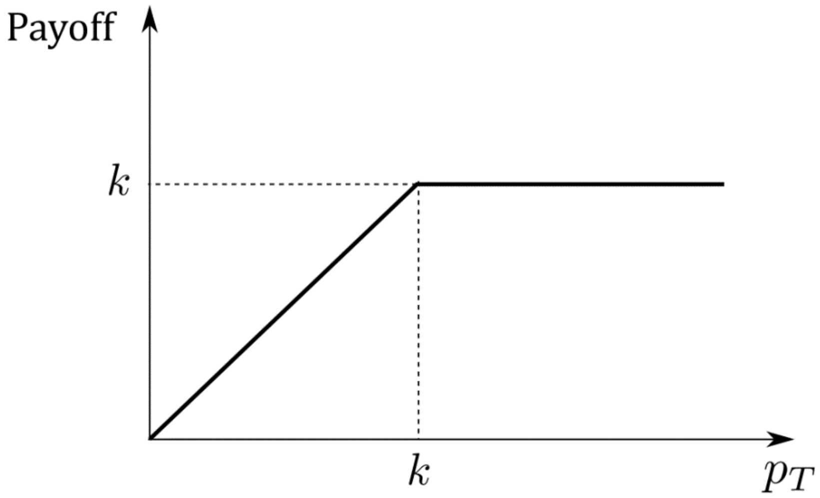
Buyer: Buyer of a callable forward with expiration date T and strike price k buys contract at $t < T$ for amount x of underlying commodity

Obligations and payoffs: At $t < T$ buyer pays seller the price of the callable forward, at T seller pays buyer $\min(p_T, k) \cdot x$, where p_T is the spot price of the underlying commodity

The function of callable forward contracts

Curtail the provision of a commodity to the buyer of the contract when $p_T \geq k$:

- If $p_T \leq k$, buyer receives p_T from seller and can buy the commodity in the spot market
- If $p_T > k$, buyer receives k



Callable forward contracts

The price of callable forward contracts

The virtues of callable forward contracts

Price of callable forward contracts

Define

$$Q_t(p) = \mathbb{P}[p_T \leq p | \xi_{[t]}]$$

where $\xi_{[t]}$ is information at time t

Assuming density of $Q_t(p)$ exists,

$$q_t(p) = \frac{dQ_t(p)}{dp}$$

Price of forward/callable forward at time t:

$$f_t = \mathbb{E}[f_T | \theta_t] = \int_0^\infty p \cdot q_t(p) dp \quad (1)$$

$$j_t(k) = \mathbb{E}[j_T(k) | \theta_t] = \int_0^\infty \min(p, k) \cdot q_t(p) dp \quad (2)$$

$q_t(p)$ implies $j_t(k)$ and vice versa

Integrating by parts:

$$j_t(k) = k - \int_0^k Q_t(p)dp = \int_0^k \big(1 - Q_t(p)\big)dp \quad (3)$$

Differentiating with respect to k:

$$\frac{dj_t(k)}{dk} = 1 - Q_t(k) \quad (4)$$

Differentiating again with respect to k:

$$q_t(k) = -\frac{d^2 j_t(k)}{dk^2} \quad (5)$$

Properties of callable forward price

- $j_t(k)$ is non-decreasing, concave in k
  - Proof: follows from equations (4), (5)
  - Intuition: higher strike price increases payoff for holder
- $j_t(k) \leq k$ for all k
  - Proof: follows from equation (3)
  - Intuition: callable forward cannot pay more than k
- $\lim_{k \to \infty} j_t(k) = f_t$
  - Proof: follows from equations (1), (2)
  - Intuition: as k increases, likelihood of $p_T \leq k$ decreases

Example

- Consider a market with the following prices:
  - 1000 $/MWh for hours 1-20
  - 880.04 $/MWh for hour 21
  - 160 $/MWh for hours 22-328
  - 120.06 $/MWh for hour 329
  - 80 $/MWh for hours 330-1752
  - 25.21 $/MWh for hour 1753
  - 25 $/MWh for hours 1754-7576
  - 10.81 $/MWh for hour 7577
  - 6.5 $/MWh for hours 7578-8760

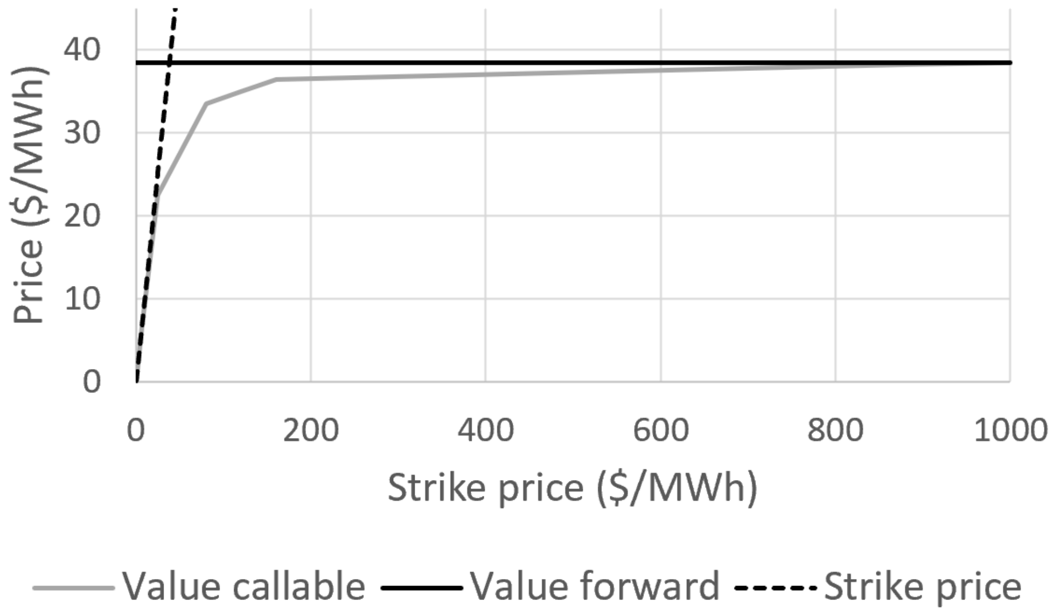
Prices of derivatives in the market of the example

- Price of forward contract:
  $$f_t = (20 \cdot 1000 + 1 \cdot 880.04 + 307 \cdot 160 + 1 \cdot 120.06 + 1423 \cdot 80 + 1 \cdot 25.21 + 5823 \cdot 25 + 1 \cdot 10.81 + 1183 \cdot 6.5)/8760 = 38.5 \text{ \$/MWh}$$

- Price of callable forward contract with strike price of 300 \$/MWh
  $$j_t(300) = (20 \cdot 300 + 1 \cdot 300 + 307 \cdot 160 + 1 \cdot 120.06 + 1423 \cdot 80 + 1 \cdot 25.21 + 5823 \cdot 25 + 1 \cdot 10.81 + 1183 \cdot 6.5)/8760 = 36.84 \text{ \$/MWh}$$

- Price of call option with strike price of 300 \$/MWh:
  $$f_t - j_t(k) = 1.66 \text{ \$/MWh}$$

Price of forward contract for different strike prices



We confirm the three properties of slide 50

Callable forward contracts

The price of callable forward contracts

The virtues of callable forward contracts

Virtues of callable forward contracts

- Useful for integrating demand response
- Consumers self-select the «right» contract
- Callable forward contracts can be traded

Integration of demand response

Mutual benefits from callable forward contracts for loads and system operator:

- Loads with valuation v *always* receive full value of power supply, regardless of real-time price of electricity, by selecting $k = v$
  - If $p_T \leq v$, loads consume power
  - If $p_T > v$, loads receive compensation $k = v$ (equivalent to consuming power)
- System operator receives information about demand function, beneficial for system planning

Consumer self-selection

Assuming risk-neutral consumers, callable forward contracts priced according to the following payoff:

$$\mathbb{E}\left[B_t(k)\middle|\xi_{[t]}\right] = Q_t(k) \cdot v + \left(1 - Q_t(k)\right) \cdot k - j_t(k)$$
$$= k + Q_t(k) \cdot (v - k) - j_t(k) \quad (6)$$

where $B_t(k)$ is consumer benefit

From equation (4) it follows that

$$\frac{d\mathbb{E}\left[B_t(k)\middle|\xi_{[t]}\right]}{dk} = 1 - \frac{dj_t(k)}{dk} - Q_t(k) + (v - k) \cdot q_t(k) = (v - k) \cdot q_t(k) \quad (7)$$

Suppose that $q_t(k) > 0$ for all $k > 0$

- $k = v$ is the unique solution that maximizes expected consumer benefit
  - $\frac{d\mathbb{E}\left[B_t(k)\middle|\xi_{[t]}\right]}{dk} = 0$ για $k = v$
  - $\frac{d\mathbb{E}\left[B_t(k)\middle|\xi_{[t]}\right]}{dk} > 0$ για $k < v$
  - $\frac{d\mathbb{E}\left[B_t(k)\middle|\xi_{[t]}\right]}{dk} < 0$ για $k > v$
- Buying callable forward contracts is better than not buying them
  - From equation (6), expected payoff for $k = v$ is $v - j_t(v)$
  - From equation (3) and $q_t(k) > 0$, $v - j_t(v) > 0$

Modeling risk aversion

Value at risk and conditional value at risk

Worst-case characterization of coherent risk measures

Back-propagation

Other ways of representing risk aversion

Risk measure

A **risk measure** is a mapping from a real-valued random variable $\xi: \Omega \to \mathbb{R}$ to a real number

Intuition: risk measures score lotteries accounting for the risk of the lotteries

Example: forward contract as a lottery

Consider a forward contract for future delivery of one MW of electricity during the winter

States of the world: $\Omega = \{\text{Cold}, \text{Hot}\}$

- Cold ⇒ electric heating ⇒ high electricity prices (100 \$/MWh)
- Hot ⇒ no electric heating ⇒ low electricity prices (50 \$/MWh)

The forward contract is an obligation of the seller to pay the price of electricity on the date of delivery:

- $\xi(\text{Cold}) = 100\ \$/\text{MWh}$
- $\xi(\text{Hot}) = 50\ \$/\text{MWh}$

Example: call option as a lottery

Consider an agent that has sold a call option with a strike price of $k = 70\ \$/\mathrm{MWh}$

- $\xi(\mathrm{Cold}) = \max(100 - 70, 0) = 30\ \$/\mathrm{MWh}$
- $\xi(\mathrm{Hot}) = \max(50 - 70, 0) = 0\ \$/\mathrm{MWh}$

Example: expected value as a risk measure

Expected value $\mathcal{R}(\xi) = \mathbb{E}[\xi]$ is the most commonly used risk measure

Returning to the previous examples:

- Forward contract: $\mathcal{R}(\xi) = 75\ \$/\mathrm{MWh}$
- Call option: $\mathcal{R}(\xi) = 15\ \$/\mathrm{MWh}$

Example: worst-case payoff as a risk measure

Consider the risk measure $\mathcal{R}(\xi) = \max_{\omega \in \Omega} \xi(\omega)$, i.e. the worst possible payoff

- Forward contract: $\mathcal{R}(\xi) = 100 \text{ \$/MWh}$
- Call option: $\mathcal{R}(\xi) = 30 \text{ \$/MWh}$

Coherent risk measure

$\mathcal{R}(\cdot)$ is a **coherent risk measure (CRM)** if the following hold:

1. Subadditivity: $\mathcal{R}(\xi + \zeta) \leq \mathcal{R}(\xi) + \mathcal{R}(\zeta)$ for any random variables ξ and ζ
   - Intuition: pooling risk is good
2. Positive homogeneity of degree one: $\mathcal{R}(\lambda \cdot \xi) = \lambda \cdot \mathcal{R}(\xi)$ for all $\lambda \geq 0$
   - Intuition: discounting costs discounts risk
3. Monotonoicity: $\mathcal{R}(\xi) \leq \mathcal{R}(\zeta)$ whenever $\xi \lesssim \zeta$, where $\lesssim$ denotes **first-order stochastic dominance**, i.e. $\mathbb{P}[\xi \leq t] \geq \mathbb{P}[\zeta \leq t]$, for all $t \in \mathbb{R}$
   - Intuition: lower costs imply lower risk
4. Translation invariance: $\mathcal{R}(\xi + t) = \mathcal{R}(\xi) + t$ for any $t \in \mathbb{R}$
   - Intuition: fixed costs add a fixed amount of risk

Example: stochastic dominance of electricity prices

Consider the price distribution of the example of slide 61

And consider the electricity price ζ of another market for which

- The price is 50 \$/MWh with probability 0.25
- The price is 120 \$/MWh with probability 0.75

We have that $\xi \precsim \zeta$ (you should check this)

Some risk measures that are and that are not coherent

Some coherent risk measures:

- Expected value
- Worst-case payoff

Some risk measures that are not coherent:

- Value at risk
- Markowitz risk measure

Convex risk measures

The function $\mathcal{R}$ is a **convex risk measure** if it satisfies conditions 1-3 of the definition of coherent risk measures

Subadditivity and positive homoegeneity (conditions 1 and 2) imply that $\mathcal{R}$ is convex

- Intuition: the marginal cost of risk is increasing

Modeling risk aversion

Value at risk and conditional value at risk

Worst-case characterization of coherent risk measures

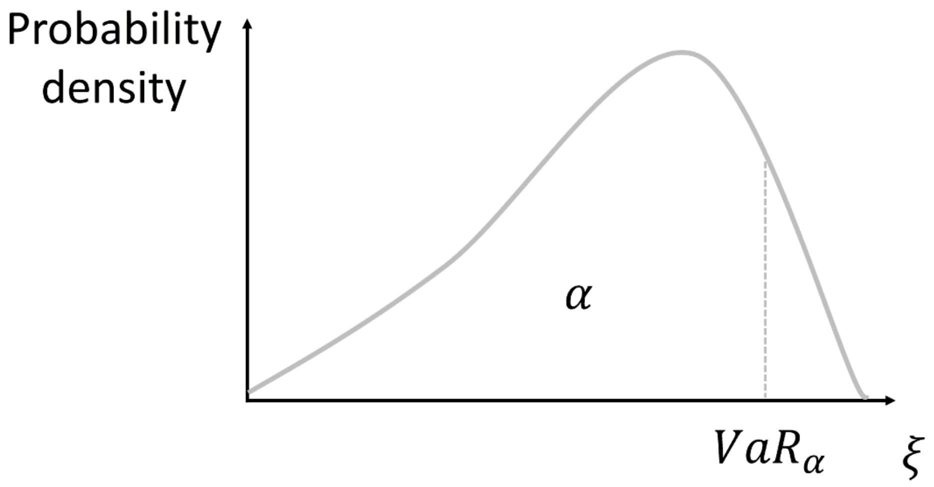
Back-propagation

Other ways of representing risk aversion

Value at risk

Value at risk (VaR) is the greatest loss in portfolio value that can occur with probability α:

$$VaR_\alpha(\xi) = \min\{t|\mathbb{P}[\xi \leq t] \geq a\}$$



Example: value at risk

Consider the forward contract of slide 61

- $VaR_{0.1}(\xi)$ = 50 \$/MWh
  - Intuition: if an investor observes the payoff of the forward contract that it is obliged to settle for 1000 market outcomes and picks the best 100 among them, then the cost it is required to pay off can be as high as 50 \$/MWh
- $VaR_{0.9}(\xi) = 100$ \$/MWh
  - Intuition: state it

Drawbacks of value at risk

- Highly sensitive/unstable with respect to market data
  - Rockafellar: "*This degree of instability is distressing for a measure of risk on which enormous sums might be riding*"

- Not a coherent risk measure (not subadditive)

Example: value at risk is not subadditive

Consider two possible states of the world: $\Omega = \{1,2\}$, with equal probabilities

And consider two random variables:

- $\xi(1) = 10, \xi(2) = 100$
- $\zeta(1) = 100, \zeta(2) = 10$

We have that $VaR_{0.1}(\xi) = 10, VaR_{0.1}(\zeta) = 10$

But we have that $VaR_{0.1}(\xi + \zeta) = 110 > VaR_{0.1}(\xi) + VaR_{0.1}(\zeta)$

Conditional value at risk

Conditional value at risk (CVaR) is the expectation of losses, conditional on losses being greater than VaR:

$$CVaR_\alpha(\xi) = \mathbb{E}_{P_\alpha}[\xi]$$

where

$$P_\alpha(t) = \begin{cases} 0, \text{if } t < VaR_\alpha(\xi) \\ \dfrac{\mathbb{P}[\xi \leq t] - \alpha}{1 - \alpha}, \text{if } t \geq VaR_\alpha(\xi) \end{cases}$$

Advantages of CVaR relative to VaR

- More stable with respect to data
- Coherent risk measure
- Can be represented as a linear program

Representation of CVaR as a linear program (Rockafellar and Uryasev [3])

We have

$$CVaR_\alpha(\xi) = \min_t\{t + \frac{1}{1-\alpha}\mathbb{E}_P[(\xi - t)^+]\}$$

where the optimal solution equals VaR_α

Example: computation of CVaR

Consider the payoffs in the table

Cost ($)	Probability (%)
1000	4
0	10
-1000	12
-2000	14
-3000	60

We will show that

- $CVaR_{0.96} = \$1000$
- $CVaR_{0.86} = \frac{4}{14} \cdot (1000) + \frac{10}{14} \cdot 0 = \$285.7$

Computation by hand

- For $CVaR_{0.96}$:
  - The 4% least favorable outcomes correspond to the unique realization where cost equals $1000
  - The conditional probability of this event occurring is 100%

- For $CVaR_{0.86}$:
  - If the least 14% favorable outcomes occur, this corresponds to the outcomes with cost $1000 and $0
  - The conditional distribution then assigns a probability of (4/14) to the outcome with cost $1000, and a probability of (10/14) to the outcome with cost $0

Computation as a linear program

Using the result of slide 76:

$$\min_{t,y} t + \frac{1}{1-0.86}(0.04 \cdot y_1 + 0.1 \cdot y_2 + 0.12 \cdot y_3 + 0.14 \cdot y_4 + 0.6 \cdot y_5)$$

$$y_1 \geq 1000 - t$$
$$y_2 \geq 0 - t$$
$$y_3 \geq -1000 - t$$
$$y_4 \geq -2000 - t$$
$$y_5 \geq -3000 - t$$
$$y \geq 0$$

If our solver computes an objective function of 285.7 and an optimal value t equal to 0, what can we conclude about $CVaR_{0.86}$ and $VaR_{0.86}$?

Modeling risk aversion

Value at risk and conditional value at risk

Worst-case characterization of coherent risk measures

Back-propagation

Other ways of representing risk aversion

Worst-case characterization of coherent risk measures

$\mathcal{R}$ is a coherent risk measure if and only if there exists a class of probability measures $\mathcal{M}$ such that $\mathcal{R}(\xi)$ equals the highest expectation of ξ with respect to members of this class:

$$\mathcal{R}(\xi) = \max_{q \in \mathcal{M}} \mathbb{E}_P[\xi] = \max_{q \in \mathcal{M}} \sum_{\omega \in \Omega} q_\omega \cdot \xi(\omega)$$

The vector $\bar{q}$ that maximizes this expression is the **risk-adjusted probability measure**

Example: CVaR

Consider the following class of probability measures:

$$\mathcal{M} = \left\{ q : q_\omega \leq \frac{P_\omega}{\alpha}, \sum_{\omega \in \Omega} q_\omega = 1, q \geq 0 \right\}$$

Interpretation: we allow ourselves to redistribute the probability of all outcomes by increasing the original probabilities P_ω by a factor $1/\alpha$

If our goal is to maximize the damage caused by $\xi(\omega)$, we "push" as much probability as possible to the higher values of $\xi(\omega) \Rightarrow$ distribution of slide 74

So CVaR is a coherent risk measure with risk-adjusted probability that of slide 74

Subgradient of risk-adjusted payoff

The vector $\bar{q}$ is a subgradient of $\mathcal{R}(\xi)$

Moreover, the subgradient of a risk measure with respect to a parameter a can be derived using the chain rule:

$$\frac{\partial \mathcal{R}(\xi)}{\partial a} = \sum_{\omega \in \Omega} \frac{\partial \mathcal{R}}{\partial \xi} \frac{\partial \xi}{\partial a} = \mathbb{E}_{\bar{q}} \left[\frac{\partial \xi}{\partial a} \right]$$

Example: sugradient of risk-adjusted payoff

Consider an agent that sells a MW of electricity in a forward market

The cost of settling the forward contract in real time is

$$\xi(\omega) = \lambda^{RT}(\omega) \cdot \alpha$$

Suppose that prices are uniformly distributed between $\{10,20, \ldots, 1000\}$ \$/MWh

And suppose that the risk aversion of the agent is $CVaR_{0.8}$

Example (continued)

The risk-adjusted probability measure increases the probability of the worst outcomes by a factor of 1.25:

$$\bar{q} = \{0,0,0.125, \ldots, 0.125\}$$

Proof: slide 82

The subgradient of the payoff ξ with respect to the forward position a is $\lambda^{RT}(\omega)$:

$$\frac{\partial \xi}{\partial a} = \lambda^{RT}$$

Intuition: selling an extra MW in the forward day-ahead market costs the agent $\lambda^{RT}(\omega)$ if outcome ω materializes

Παράδειγμα (συνέχεια)

The risk-adjusted cost of selling one more MW in the forward market is

$$\frac{\partial \mathcal{R}(\xi)}{\partial a} = \mathbb{E}_{\bar{q}}\left[\frac{\partial \xi}{\partial a}\right] = \mathbb{E}_{\bar{q}}\left[\lambda^{RT}\right] = 0.125 \cdot (30 + 40 + \cdots + 100) = \$65$$

The expected real-time price is $55

The risk aversion of the agent means that it assigns a cost of $65 for selling one MW in the forward market (higher)

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Trick question

The day-ahead market is the most important electricity market, because this is where the greatest volumes are transacted

And the real-time market is of secondary importance, because that is where corrections take place with a small amount of traded volumes

Right or wrong?

Back-propagation

Back-propagation is the process by which forward prices are formed as a function of the distribution of real-time prices

And not the opposite!

Intuition of back-propagation

We have the tools to prove back-propagation (and will do so next)

But the intuition is the following:

- If the forward price of electricity is higher than the expected real-time price, then
  - Agents sell forwards and close their position in real time, thereby achieving a positive expected profit
  - The sale of forward contracts exerts downward pressure on forward prices (and has the contrary effect on real-time prices)
  - $\Rightarrow$ alignment of forward prices to expected real-time prices
- Develop the argument in the opposite case

Quantitative argument

Consider an agent that decides how much electricity to sell in the forward day-ahead market

And assume **virtual trading**:

- We can sell even if we do not own generating assets
- We can buy even if we are not serving loads

Profit maximization:

$$\max_a \lambda^{DA} \cdot a - \mathcal{R}\big(\xi(a)\big)$$

where $\xi(a) = \lambda^{RT} \cdot a$

First-order optimality condition:

$$\lambda^{DA} = \frac{\partial \mathcal{R}}{\partial a} = \mathbb{E}_{\bar{q}}\left[\lambda^{RT}\right]$$

Example

We return to the example of slide 84

If the price in the day-ahead energy market exceeds 65 $/MWh, what position would the agent take in the day-ahead market (short or long)?

If the price in the day-ahead energy market is less than 65 $/MWh, what position would the agent take in the day-ahead market (short or long)?

If all agents have the same attitude towards risk, what is the equilibrium price of the day-ahead market?

So what?

Back-propagation proves the central role of balancing markets/real-time markets in electricity market design

This central role was not recognized adequately in the original design of the European market:

- There was disproportionate emphasis on the design of the day-ahead market
- And the real-time market was considered of secondary importance, and not designed carefully

We are in the midst of important reforms in the European real-time electricity markets (TERRE, MARI, PICASSO, IGCC, imbalance settlement)

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Other ways of representing risk aversion

Utility functions

Convex **utility functions** are a common way for representing risk aversion in eocnomics

$$\mathcal{R}(\xi) = \mathbb{E}\left[U\left(\xi(\omega)\right)\right]$$

where U is a convex utility function $U: \mathbb{R} \to \mathbb{R}$

Example

Consider a lottery ξ that requires its owner to pay \$100 or -\$100 with equal probability

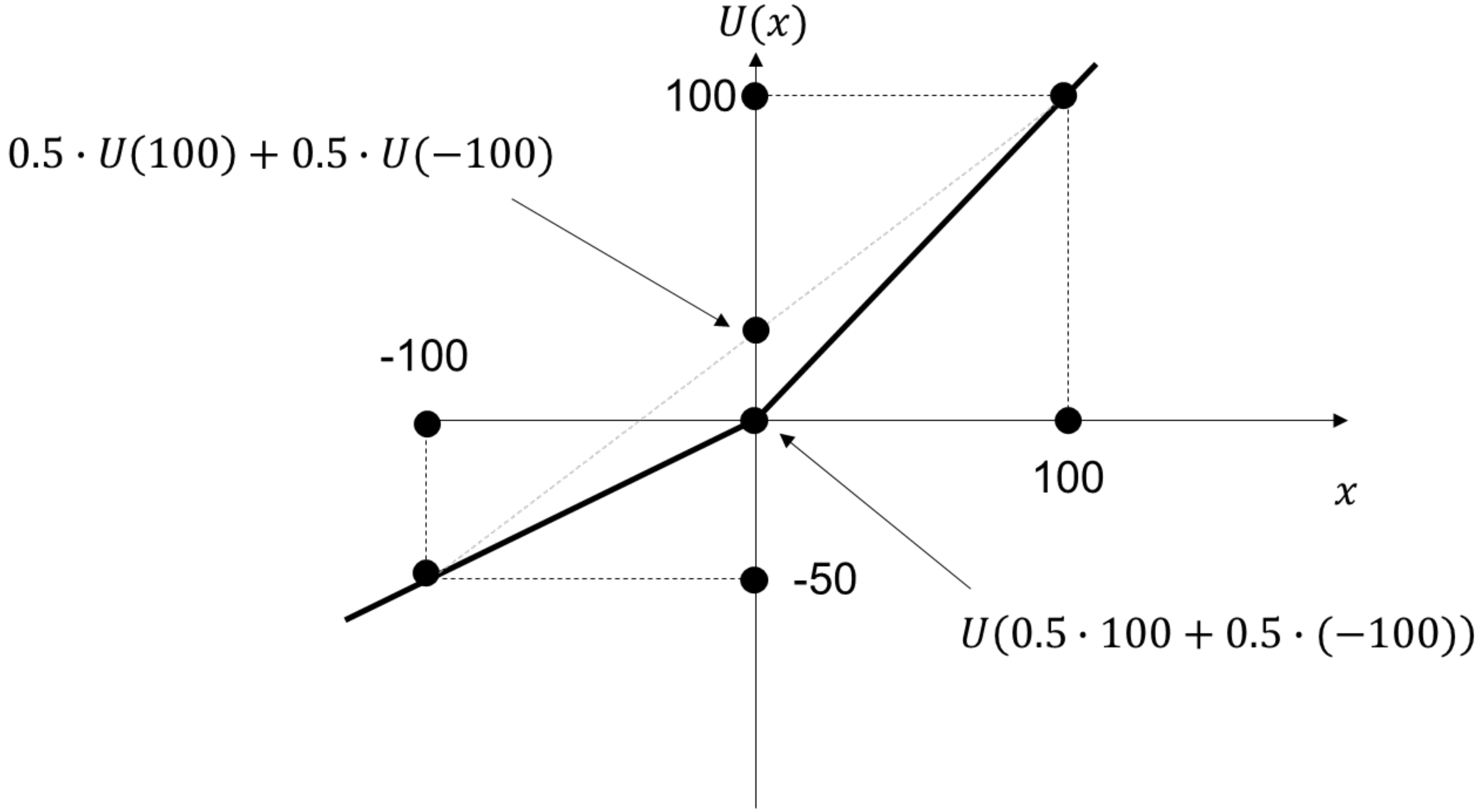
And consider the alternative ζ of a certain payment of \$0

Consider a utility function $\max(0.5 \cdot x, x)$

We have

- $\mathcal{R}(\xi) = 0.5 \cdot (100 - 50) = 25$
- $\mathcal{R}(\zeta) = 0$

Since $\mathcal{R}(\xi) \geq \mathcal{R}(\zeta)$, the certain payment is preferable



$U(x)$
100
$0.5 \cdot U(100) + 0.5 \cdot U(-100)$
-100
100
x
-50
$U(0.5 \cdot 100 + 0.5 \cdot (-100))$

Markowitz risk measure

The **Markowitz risk measure** is defined as:

$$\mathcal{R}(\xi) = \mathbb{E}[\xi] + \beta \cdot var(\xi)$$

where β is a fixed parameter and $var(\xi)$ is the variance of ξ

Example

Consider the lottery of slide 92

We have

$$\mathbb{E}[\xi] = 0.5 \cdot 100 + 0.5 \cdot (-100) = 0$$
$$var(\xi) = 0.5 \cdot (100 - 0)^2 + 0.5 \cdot (-100 - 0)^2 = 10000$$

For $\beta = 0.05$ we have

$$\mathcal{R}(\xi) = 0 + 0.05 \cdot 10000 = 500$$

References

[1] A. Papavasiliou, Optimization Models in Electricity Markets, Cambridge University Press

https://www.cambridge.org/highereducation/books/optimization-models-in-electricity-markets/0D2D36891FB5EB6AAC3A4EFC78A8F1D3#overview

[2] Hogan, William W. "Contract networks for electric power transmission." Journal of regulatory economics 4.3 (1992): 211-242.

[3] Rockafellar, R. Tyrrell, and Stanislav Uryasev. "Optimization of conditional value-at-risk." Journal of risk 2 (2000): 21-42.