Using AMPL with Gurobi to Apply Optimization Models Efficiently and Reliably

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Outline

Motivation

- The optimization modeling cycle
- Optimization modeling languages
- AMPL and Gurobi

Introductory example

The AMPL company

AMPL's users

- Commercial
- Government
- Research & teaching

Future directions

The Optimization Modeling Cycle

Steps

- Communicate with problem owner
- Build model
- Build datasets
- Generate optimization problems
- Feed problems to solvers ***(Gurobi)***
- Solve
- Process results for analysis & reporting
- ***Repeat!***

Goals

- Do this quickly and reliably
- Get results before client loses interest
- Deploy for application

Optimization in 1980

"We do not feel that the linear programming user's most pressing need over the next few years is for a new optimizer that runs twice as fast on a machine that costs half as much (although this will probably happen). Cost of optimization is just not the dominant barrier to LP model implementation. The process required to manage the data, formulate and build the model, report on and analyze the results costs far more, and is much more of a barrier to effective use of LP, than the cost/performance of the optimizer."

Krabek, Sjoquist, Sommer,
"The APEX Systems: Past and Future."
SIGMAP Bulletin 29 (April 1980) 3-23.

Optimization Modeling Languages

Two forms of an optimization problem

- Modeler's form
  - Mathematical description, easy for people to work with
- Algorithm's form
  - Explicit data structure, easy for solvers to compute with

Idea of a modeling language

- A computer-readable modeler's form
  - You write optimization problems in a modeling language
  - Computers translate to algorithm's form for solution

Advantages of a modeling language

- Faster modeling cycles
- More reliable modeling and maintenance

Algebraic Modeling Languages

Formulation concept

- Define data in terms of sets & parameters
  - Analogous to database keys & records
- Define decision variables
- Minimize or maximize a function of decision variables
- Subject to equations or inequalities
  that constrain the values of the variables

Advantages

- Familiar
- Powerful
- Implemented

The AMPL Modeling Language

Features

- Algebraic modeling language
- Variety of data sources
- Connections to all solver features
- Interactive and scripted control

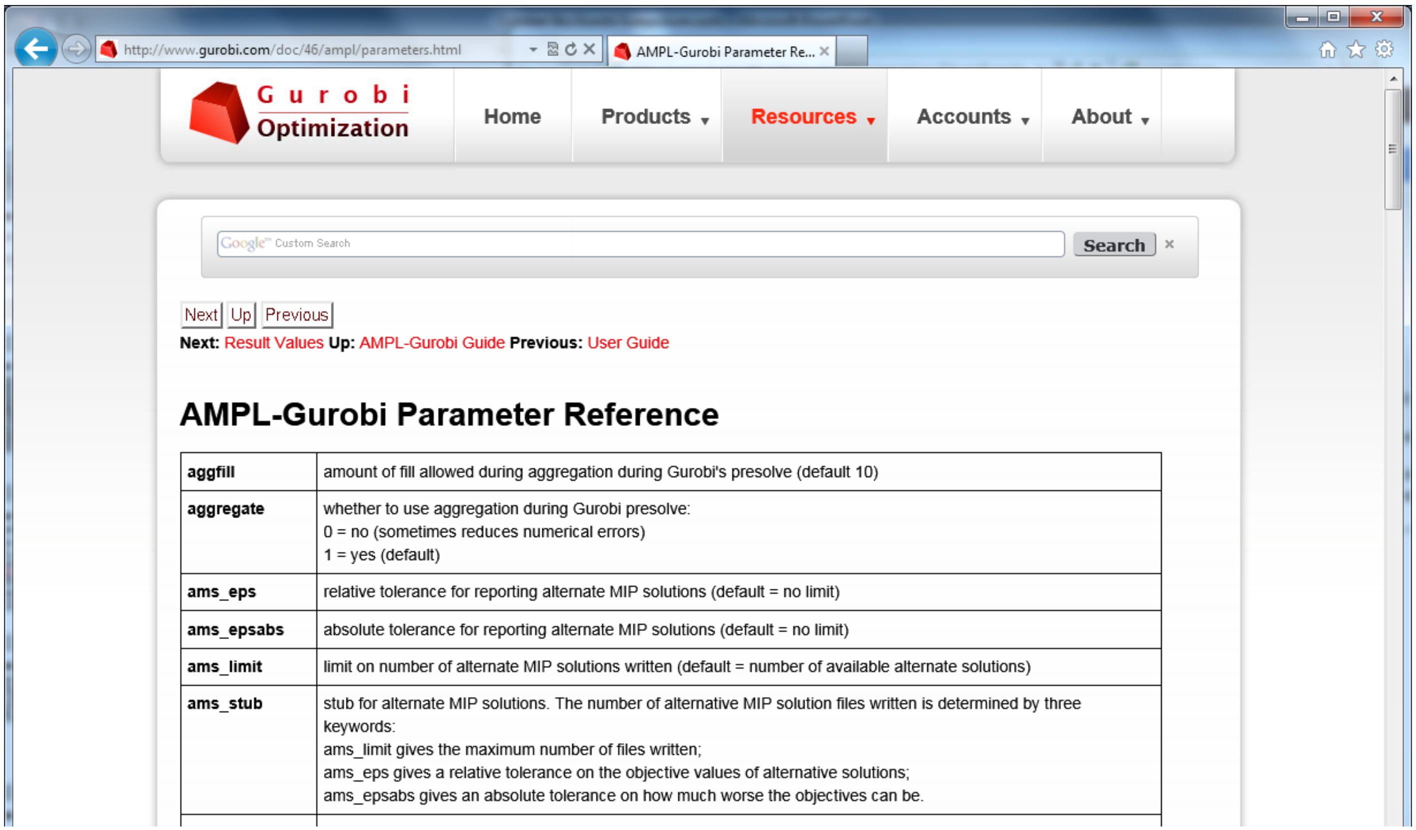
Advantages

- Powerful, general expressions
- Natural, easy-to-learn design
- Efficient processing scales well with problem size

AMPL with Gurobi

Features

- Detection of all supported problem types
- Access to all algorithm & display options



Next: Result Values **Up:** AMPL-Gurobi Guide **Previous:** User Guide

AMPL-Gurobi Parameter Reference

aggfill	amount of fill allowed during aggregation during Gurobi's presolve (default 10)
aggregate	whether to use aggregation during Gurobi presolve: 0 = no (sometimes reduces numerical errors) 1 = yes (default)
ams_eps	relative tolerance for reporting alternate MIP solutions (default = no limit)
ams_epsabs	absolute tolerance for reporting alternate MIP solutions (default = no limit)
ams_limit	limit on number of alternate MIP solutions written (default = number of available alternate solutions)
ams_stub	stub for alternate MIP solutions. The number of alternative MIP solution files written is determined by three keywords: ams_limit gives the maximum number of files written; ams_eps gives a relative tolerance on the objective values of alternative solutions; ams_epsabs gives an absolute tolerance on how much worse the objectives can be.

Introductory Example

Multicommodity transportation . . .

- ❖ Products available at factories
- ❖ Products needed at stores
- ❖ Plan shipments at lowest cost

. . . with practical restrictions

- ❖ Cost has fixed and variables parts
- ❖ Shipments cannot be too small
- ❖ Factories cannot serve too many stores

Multicommodity Transportation

Given

- O Set of origins (factories)
- D Set of destinations (stores)
- P Set of products

and

- a_{ip} Amount available, for each $i \in O$ and $p \in P$
- b_{jp} Amount required, for each $j \in D$ and $p \in P$
- l_{ij} Limit on total shipments, for each $i \in O$ and $j \in D$
- c_{ijp} Shipping cost per unit, for each $i \in O$, $j \in D$, $p \in P$
- d_{ij} Fixed cost for shipping any amount from $i \in O$ to $j \in D$
- s Minimum total size of any shipment
- n Maximum number of destinations served by any origin

Mathematical Formulation

Determine

X_{ijp} Amount of each $p \in P$ to be shipped from $i \in O$ to $j \in D$

Y_{ij} 1 if any product is shipped from $i \in O$ to $j \in D$
0 otherwise

to minimize

$\sum_{i \in O} \sum_{j \in D} \sum_{p \in P} c_{ijp} X_{ijp} + \sum_{i \in O} \sum_{j \in D} d_{ij} Y_{ij}$

Total variable cost plus total fixed cost

Multicommodity Transportation

Mathematical Formulation

Subject to

$\sum_{j \in D} X_{ijp} \leq a_{ip}$ for all $i \in O$, $p \in P$

Total shipments of product p out of origin i must not exceed availability

$\sum_{i \in O} X_{ijp} = b_{jp}$ for all $j \in D$, $p \in P$

Total shipments of product p into destination j must satisfy requirements

Mathematical Formulation

Subject to

$\sum_{p\in P} X_{ijp} \leq l_{ij} Y_{ij}$ for all $i \in O,\ j \in D$

When there are shipments from origin i to destination j, the total may not exceed the limit, and Y_{ij} must be 1

$\sum_{p\in P} X_{ijp} \geq s Y_{ij}$ for all $i \in O,\ j \in D$

When there are shipments from origin i to destination j, the total amount of shipments must be at least s

$\sum_{j\in D} Y_{ij} \leq n$ for all $i \in O$

Number of destinations served by origin i must be as most n

Multicommodity Transportation

AMPL Formulation

Symbolic data

```
set ORIG;   # origins
set DEST;   # destinations
set PROD;   # products

param supply {ORIG,PROD} >= 0;  # availabilities at origins
param demand {DEST,PROD} >= 0;  # requirements at destinations
param limit {ORIG,DEST} >= 0;   # capacities of links

param vcost {ORIG,DEST,PROD} >= 0; # variable shipment cost
param fcost {ORIG,DEST} > 0;       # fixed usage cost

param minload >= 0;             # minimum shipment size
param maxserve integer > 0;     # maximum destinations served
```

Multicommodity Transportation

AMPL Formulation

Symbolic model: variables and objective

```
var Trans {ORIG,DEST,PROD} >= 0;   # actual units to be shipped
var Use {ORIG, DEST} binary;       # 1 if link used, 0 otherwise

minimize Total_Cost:
   sum {i in ORIG, j in DEST, p in PROD} vcost[i,j,p] * Trans[i,j,p]
 + sum {i in ORIG, j in DEST} fcost[i,j] * Use[i,j];
```

$$\sum_{i \in O} \sum_{j \in D} \sum_{p \in P} c_{ijp} X_{ijp} + \sum_{i \in O} \sum_{j \in D} d_{ij} Y_{ij}$$

Multicommodity Transportation

AMPL Formulation

Symbolic model: constraint

```
subject to Supply {i in ORIG, p in PROD}:
   sum {j in DEST} Trans[i,j,p] <= supply[i,p];
```

$$\sum_{j \in D} X_{ijp} \leq a_{ip}, \text{ for all } i \in O, p \in P$$

AMPL Formulation

Symbolic model: constraints

```
subject to Supply {i in ORIG, p in PROD}:
   sum {j in DEST} Trans[i,j,p] <= supply[i,p];

subject to Demand {j in DEST, p in PROD}:
   sum {i in ORIG} Trans[i,j,p] = demand[j,p];

subject to Multi {i in ORIG, j in DEST}:
   sum {p in PROD} Trans[i,j,p] <= limit[i,j] * Use[i,j];

subject to Min_Ship {i in ORIG, j in DEST}:
   sum {p in PROD} Trans[i,j,p] >= minload * Use[i,j];

subject to Max_Serve {i in ORIG}:
   sum {j in DEST} Use[i,j] <= maxserve;
```

Multicommodity Transportation

AMPL Formulation

Explicit data independent of symbolic model

```
set ORIG := GARY CLEV PITT ;
set DEST := FRA DET LAN WIN STL FRE LAF ;
set PROD := bands coils plate ;

param supply (tr):  GARY   CLEV   PITT :=
            bands    400    700    800
            coils    800   1600   1800
            plate    200    300    300 ;

param demand (tr):
           FRA   DET   LAN   WIN   STL   FRE   LAF :=
   bands   300   300   100    75   650   225   250
   coils   500   750   400   250   950   850   500
   plate   100   100     0    50   200   100   250 ;

param limit default 625 ;

param minload := 375 ;
param maxserve := 5 ;
```

AMPL Formulation

Explicit data (continued)

```
param vcost :=

 [*,*,bands]:  FRA  DET  LAN  WIN  STL  FRE  LAF :=
       GARY    30   10    8   10   11   71    6
       CLEV    22    7   10    7   21   82   13
       PITT    19   11   12   10   25   83   15

 [*,*,coils]:  FRA  DET  LAN  WIN  STL  FRE  LAF :=
       GARY    39   14   11   14   16   82    8
       CLEV    27    9   12    9   26   95   17
       PITT    24   14   17   13   28   99   20

 [*,*,plate]:  FRA  DET  LAN  WIN  STL  FRE  LAF :=
       GARY    41   15   12   16   17   86    8
       CLEV    29    9   13    9   28   99   18
       PITT    26   14   17   13   31  104   20 ;

param fcost:   FRA  DET  LAN  WIN  STL  FRE  LAF :=
       GARY  3000 1200 1200 1200 2500 3500 2500
       CLEV  2000 1000 1500 1200 2500 3000 2200
       PITT  2000 1200 1500 1500 2500 3500 2200 ;
```

AMPL Solution

Model + data = problem instance to be solved

```
ampl: model multmipG.mod;
ampl: data multmipG.dat;

ampl: option solver gurobi;

ampl: solve;

Gurobi 4.6.0: optimal solution; objective 235625
404 simplex iterations
45 branch-and-cut nodes

ampl: display Use;

Use [*,*]

:     DET FRA FRE LAF LAN STL WIN    :=
CLEV   1   1   1   0   1   1   0
GARY   0   0   0   1   0   1   1
PITT   1   1   1   1   0   1   0
;
```

AMPL Solution

Examine results

```
ampl: display {i in ORIG, j in DEST}
ampl?    sum {p in PROD} Trans[i,j,p] / limit[i,j];

:      DET    FRA    FRE    LAF    LAN    STL    WIN    :=
CLEV   1      0.6    0.88   0      0.8    0.88   0
GARY   0      0      0      0.64   0      1      0.6
PITT   0.84   0.84   1      0.96   0      1      0
;

ampl: display Max_Serve.body;

CLEV   5
GARY   3
PITT   5
;

ampl: display TotalCost,
ampl?    sum {i in ORIG, j in DEST} fcost[i,j] * Use[i,j];

TotalCost = 235625
sum {i in ORIG, j in DEST} fcost[i,j]*Use[i,j] = 27600
```

Multicommodity Transportation

AMPL "Sparse" Network

Indexed over sets of pairs and triples

```
set ORIG;   # origins
set DEST;   # destinations
set PROD;   # products

set SHIP within {ORIG,DEST,PROD};

            # (i,j,p) in SHIP ==> can ship p from i to j

set LINK = setof {(i,j,p) in SHIP} (i,j);

            # (i,j) in LINK ==> can ship some products from i to j

...........

var Trans {SHIP} >= 0;   # actual units to be shipped

var Use {LINK} binary;   # 1 if link used, 0 otherwise

minimize Total_Cost:

   sum {(i,j,p) in SHIP} vcost[i,j,p] * Trans[i,j,p]

 + sum {(i,j) in LINK} fcost[i,j] * Use[i,j];
```

AMPL Scripting

Script to test sensitivity to serve limit

```
model multmipG.mod;
data multmipG.dat;

option solver gurobi;

for {m in 7..1 by -1} {
   let maxserve := m;
   solve;
   if solve_result = 'infeasible' then break;
   display maxserve, Max_Serve.body;
}
```

Multicommodity Transportation

AMPL Scripting

Run showing sensitivity to serve limit

```
ampl: include multmipServ.run;

Gurobi 4.6.0: optimal solution; objective 233150
maxserve = 7
CLEV 5   GARY 3   PITT 6

Gurobi 4.6.0: optimal solution; objective 233150
maxserve = 6
CLEV 5   GARY 3   PITT 6

Gurobi 4.6.0: optimal solution; objective 235625
maxserve = 5
CLEV 5   GARY 3   PITT 5

Gurobi 4.6.0: infeasible
```

AMPL Scripting

Script to generate n best solutions

```
param nSols default 0;
param maxSols;

model multmipG.mod;
data multmipG.dat;

set USED {1..nSols} within {ORIG,DEST};

subject to exclude {k in 1..nSols}:

   sum {(i,j) in USED[k]} (1-Use[i,j]) +
   sum {(i,j) in {ORIG,DEST} diff USED[k]} Use[i,j] >= 1;

option solver gurobi;

repeat {
   solve;
   display Use;
   let nSols := nSols + 1;
   let USED[nSols] := {i in ORIG, j in DEST: Use[i,j] > .5};
} until nSols = maxSols;
```

AMPL Scripting

Run showing 3 best solutions

```
ampl: include multmipBest.run;

Gurobi 4.6.0: optimal solution; objective 235625

:     DET FRA FRE LAF LAN STL WIN    :=
CLEV   1   1   1   0   1   1   0
GARY   0   0   0   1   0   1   1
PITT   1   1   1   1   0   1   0 ;

Gurobi 4.6.0: optimal solution; objective 237125

:     DET FRA FRE LAF LAN STL WIN    :=
CLEV   1   1   1   1   0   1   0
GARY   0   0   0   1   0   1   1
PITT   1   1   1   0   1   1   0 ;

Gurobi 4.6.0: optimal solution; objective 238225

:     DET FRA FRE LAF LAN STL WIN    :=

CLEV   1   0   1   0   1   1   1
GARY   0   1   0   1   0   1   0
PITT   1   1   1   1   0   1   0 ;
```

The AMPL Company

AMPL history

The AMPL team

Recent developments

- Business
- Academic

AMPL History

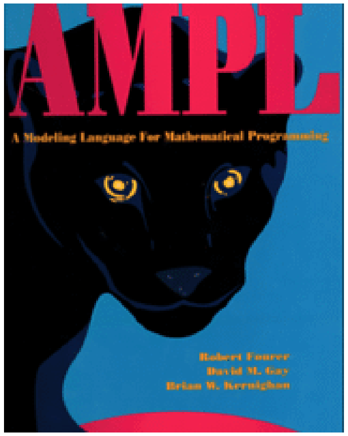
Origins

- AMPL developed at AT&T Bell Laboratories (1986)
  - Robert Fourer, David M. Gay, Brian W. Kernighan
- AMPL sold through distributors (1993)
- AMPL Optimization company formed (2002)

Writings

- "A Modeling Language for Mathematical Programming." *Management Science* **36** (1990) 519–554.
- The AMPL book

1993



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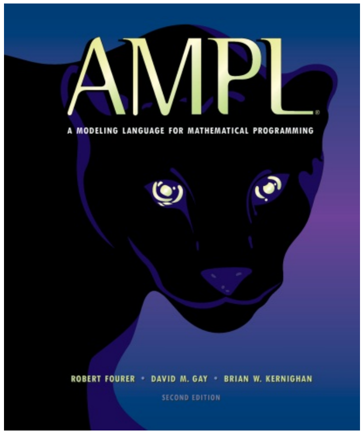


The AMPL Team

Robert Fourer

- Managing partner

David M. Gay

- Managing partner

William M. Wells

- Director of business development (joined 2010)

Victor Zverovich

- Additional expert in software development and customer support (joining in 2012)

Business Developments

AMPL intellectual property

- Full rights acquired from Alcatel-Lucent USA
  - corporate parent of Bell Laboratories
- More flexible licensing terms available

Gurobi for AMPL

- Full support from version 1.0 through 4.6 (current)

AMPL Japan distributor

- *October Sky Co., Ltd.* →

Academic Developments

Highly discounted prices for academic use

- AMPL
- Nonlinear solvers: KNITRO, MINOS, SNOPT, CONOPT

Free Gurobi for research & teaching

- 1-year license
- Renewable

Free AMPL & solvers for courses

- One-page application (`www.ampl.com/courses.html`)
- Single file for distribution to students
- Streamlined installation — no license file
- Expires after the course ends

AMPL's Users

Business

- Customer relationships
- Customer areas
- Project examples

Government

Academic

Business Customer Relationships

Internal projects

- We supply software & answer a few questions
- Company's employees build the models

Training and consulting

- Available on request

AMPL's Users

Business Customer Areas

Transportation

- Air, rail, truck

Production

- Planning
  - steel
  - automotive
- Supply chain
  - consumer products

Finance

- Investment banking
- Insurance

Natural resources

- Electric power
- Gas distribution
- Mining

Information technology

- Telecommunications
- Internet services

Consulting practices

- Management
- Industrial engineering

Business Customer Examples

Two award-winning projects

- ZARA (clothing retailing)
- Norske Skog (paper manufacturer)

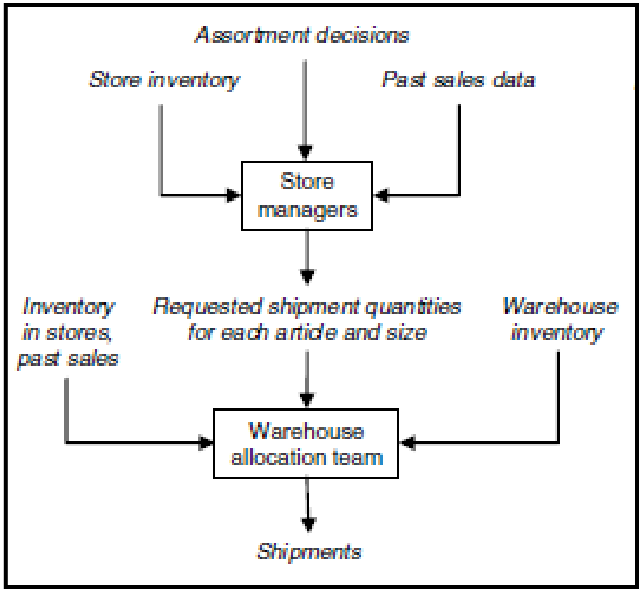
. . . finalists for Edelman Award for practice of Operations Research

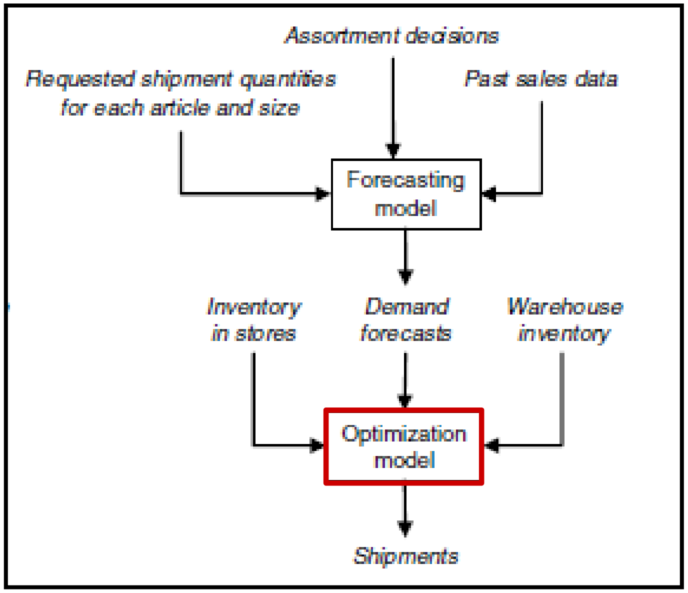
ZARA

Optimization of worldwide shipments

- Legacy process

New Process





- Piecewise-linear AMPL model with integer variables
- Run once for each product each week
- Decides how much of each size to ship to each store
- Increases sales 3-4%

ZARA's Formulation

Given

$S = S^{+} \cup S^{-}$

Set of sizes partitioned into major & regular sizes

J Set of stores

and

W_s Inventory of size s available at the warehouse

I_{sj} Inventory of size s available in store j

P_j Selling price in store j

K Aggressiveness factor (value of inventory remaining in the warehouse after the current shipments)

λ_{sj} Demand rate for size s in store j

N_{sj} Approximation set for size s in the inventory-to-sales function approximation for store j

ZARA's Formulation

Determine

x_{sj} (integer) shipment quantity of each size $s \in S$
to each store $j \in J$ for the current replenishment period

z_j (≥ 0) approximate expected sales across all sizes
in each store $j \in J$ for the current period

to maximize

$\sum_{j \in J} P_j z_j + K \sum_{s \in S} (W_s - \sum_{j \in J} x_{sj})$

Total sales plus value of items remaining in warehouse

subject to

$\sum_{j \in J} x_{sj} \leq W_s$ for all $s \in S$

Total shipments of size s
must not exceed amount available in warehouse

AMPL's Users

ZARA's Formulation

and subject to

$z_j \le (\sum_{s \in S^+} \lambda_{sj}) y_j + \sum_{s \in S^-} \lambda_{sj} v_{sj}$ for all $j \in J$

$y_j \le a_i \lambda_{sj} (I_{sj} + x_{sj} - i) + b_i \lambda_{sj}$ for all $j \in J$, $s \in S^+$, $i \in N_{sj}$

$v_{sj} \le a_i \lambda_{sj} (I_{sj} + x_{sj} - i) + b_i \lambda_{sj}$ for all $j \in J$, $s \in S^-$, $i \in N_{sj}$

$v_{sj} \le y_j$ for all $j \in J$, $s \in S^-$

Relationship between sales
and store inventory after shipments

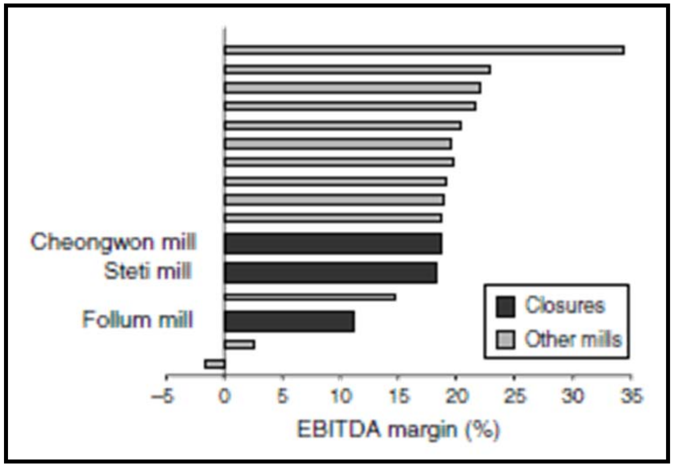
Norske Skog

Optimization of production and distribution

- Australasia
- Europe
  - 640 binary variables
  - 524,000 continuous variables
  - 33,000 constraints

Optimization of shutdown decisions worldwide

- Multiple scenarios
- Numerous sensitivity analyses
- ***Key role of AMPL models***
  - Implemented in a few weeks
  - Modified to analyze alternatives
  - Run interactively at meetings



Norske Skog's Formulation

Given sets

- N Number of mills
- M_n Set of machines at mill N
- M Number of paper machines
- J Number of products
- L Number of raw material sources
- R Number of raw materials
- K Number of customers
- P Number of recipes

Norske Skog's Formulation

Given capital parameters

l_n fixed cost of mill n running for one period (excluding machine fixed costs)

f_m fixed cost of machine m running for one period

θ_m proportion of fixed running costs saved from a temporary shutdown on machine m

q_m minimum time that machine m must be shut before savings accrue

ϕ_m amortized cost of a permanent closure of machine m

Norske Skog's Formulation

and operating parameters

g_{mjk} variable freight cost for shipping product j from machine m to customer k

a_{mjp} capacity of machine m making product j using recipe p

c_{mjp} variable cost incurred by producing one tonne of product j using recipe p on machine m

h_{mjrp} tonnes of raw material r required to make one tonne of product j using recipe p on machine m

π_{mrl} procurement, transportation, and process cost of raw material r from source l for machine m

W_{rl} supply of raw material r at source l

d_{jk} demand for product j by customer k

s_{jk} sales price for product j by customer k

Norske Skog's Formulation

Make capital decisions

δ_n 1 if mill n closes, 0 otherwise

μ_m 1 if machine m shuts down permanently, 0 otherwise

u_m time that machine m has been shut down

ξ_m 1 if machine m has been shut down long enough to accrue savings, 0 otherwise

v_m time that qualifies for savings on machine m

Norske Skog's Formulation

and operating decisions

x_{mjp} tonnes of product j made on machine m using recipe p

y_{mjk} tonnes of product j made on machine m and delivered to customer k

w_{mrl} tonnes of raw material r from source l used by machine m

σ_{mp} 1 if recipe p is used on machine m, 0 otherwise

Norske Skog's Formulation

Maximize

$$\sum_{m=1}^{M}\sum_{j=1}^{J}\left(\sum_{k=1}^{K}(s_{jk}-g_{mjk})y_{mjk}-\sum_{p=1}^{P}c_{mjp}x_{mjp}\right)$$

$$-\sum_{m=1}^{M}\sum_{l=1}^{L}\sum_{r=1}^{R}\pi_{mrl}w_{mrl}$$

$$+\sum_{m=1}^{M}\theta_m f_m v_m$$

$$-\sum_{n=1}^{N}(l_n(1-\delta_n)+\lambda_n\delta_n)$$

$$-\sum_{m=1}^{M}(f_m(1-\mu_m)+\phi_m\mu_m)$$

Income from sales,
minus raw material, production and distribution costs,
plus savings from shutdowns,
minus fixed operating and shutdown costs

Norske Skog's Formulation

Subject to

$\sum_{j=1}^{J} \sum_{p=1}^{P} \frac{x_{mjp}}{a_{mjp}} = 1 - u_m$ for $m = 1, \ldots, M$

Capacity used equals capacity available

$\sum_{k=1}^{K} y_{mjk} = \sum_{p=1}^{P} x_{mjp}$ for $j = 1, \ldots, J,\ m = 1, \ldots, M$

Amounts produced equal amounts shipped

$\sum_{m=1}^{M} y_{mjk} \leq d_{jk}$ for $j = 1, \ldots, J,\ k = 1, \ldots, K$

Amounts produced do not exceed demand

$\sum_{j=1}^{J} \sum_{p=1}^{P} h_{mjrp} x_{mjp} = \sum_{l=1}^{L} w_{mrl}$ for $m = 1, \ldots, M,\ r = 1, \ldots, R$

Raw material used equals raw material purchased

$\sum_{m=1}^{M} w_{mrl} \leq W_{rl}$ for $l = 1, \ldots, L,\ r = 1, \ldots, R$

Raw material purchased does not exceed amount available

Norske Skog's Formulation

and subject to

$\sum_{p=1}^{P} \sigma_{mp} = 1 - \mu_m$ for $m = 1, \ldots, M$

$x_{mjp} \leq a_{mjp}\sigma_{mjp}$ for $j = 1, \ldots, J,\ m = 1, \ldots, M,\ p = 1, \ldots, P$

$\delta_n \leq \mu_m$ for $m \in M_n,\ n = 1, \ldots, N$

$v_m \leq \xi_m$ for $m = 1, \ldots, M$

$v_m \leq 1 - \mu_m$ for $m = 1, \ldots, M$

$v_m \leq u_m - q_m\xi_m$ for $m = 1, \ldots, M$

Definitions of zero-one variables

Government Customers

Financial agencies

- United States
- Canada
- Sweden

U.S. departments

- Census Bureau
- Army Corps of Engineers

U.S. research centers

- Argonne National Laboratory
- Sandia National Laboratories
- Lawrence Berkeley Laboratory

Academic Customers

Research

- ❖ Over 250 university installations worldwide
- ❖ Nearly 1000 citations in scientific papers
  - ∗ engineering, science, economics, management

Teaching

- ❖ Linear & nonlinear optimization
  - ∗ Graph optimization
  - ∗ Stochastic programming
- ❖ Operations Research
- ❖ Specialized courses
  - ∗ Supply chain modeling
  - ∗ Electric power system planning
  - ∗ Transportation logistics
  - ∗ Communication network design & algorithms

Future Directions

Core development

- Further set operations
- Enhanced scripting
- More natural formulations

Interface development

- Integrated development environment
- Callable version
  - embedding in large applications
  - deployment to end users
- Support of new solver types
- Extended database support

Business expansion

- Cloud computing services

Readings

- R. Fourer, "Modeling Languages versus Matrix Generators for Linear Programming." *ACM Transactions on Mathematical Software* **9** (1983) 143–183.
- R. Fourer, D.M. Gay, B.W. Kernighan, "A Modeling Language for Mathematical Programming." *Management Science* **36** (1990) 519–554.
- R. Fourer, D.M. Gay, B.W. Kernighan, *AMPL: A Modeling Language for Mathematical Programming*. Duxbury Press, Belmont, CA (first edition 1993, second edition 2003).
- R. Fourer, "Algebraic Modeling Languages for Optimization." Forthcoming in Saul I. Gass and Michael C. Fu (eds.), *Encyclopedia of Operations Research and Management Science,* Springer (2012).
- G. Everett, A. Philpott, K. Vatn, R. Gjessing, "Norske Skog Improves Global Profitability Using Operations Research." *Interfaces* **40,** 1 (Jan–Feb 2010) 58–70.
- F. Caro, J. Gallien, M. Díaz, J. García, J.M. Corredoira, M. Montes, J.A. Ramos, J. Correa, "Zara Uses Operations Research to Reengineer Its Global Distribution Process." *Interfaces* **40,** 1 (Jan–Feb 2010) 71–84.