Locational Marginal Pricing

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Source: section 5.2, Papavasiliou [1]

Outline

- Congestion rent and congestion cost
- Competitive market model for transmission capacity
- Losses

Recall DCOPF

(DCOPF):

$$\max_{p,d,f,r} \sum_{l \in L} \int_0^{d_l} MB_l(x)\, dx - \sum_{g \in G} \int_0^{p_g} MC_g(x) dx$$

(λ_k^+):

$$f_k \leq T_k, k \in K$$

(λ_k^-):

$$-f_k \leq T_k, k \in K$$

(ψ_k):

$$f_k - \sum_{n \in N} F_{kn} \cdot r_n = 0, k \in K$$

(ρ_n):

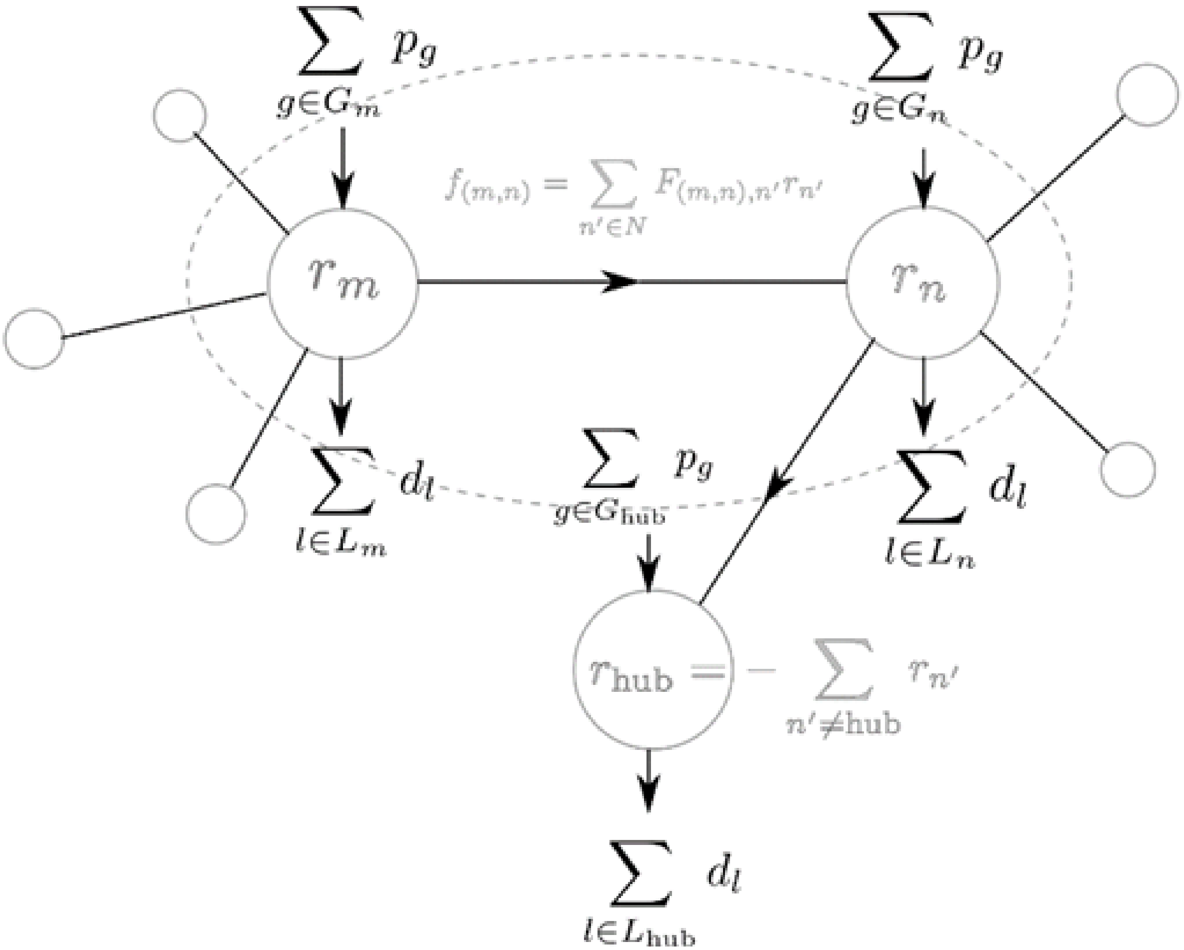
$$r_n - \sum_{g \in G_n} p_g + \sum_{l \in L_n} d_l = 0, n \in N$$

$(-\varphi)$:

$$\sum_{n \in N} r_n = 0$$

$$p_g \geq 0, g \in G$$

$$d_l \geq 0, l \in L$$

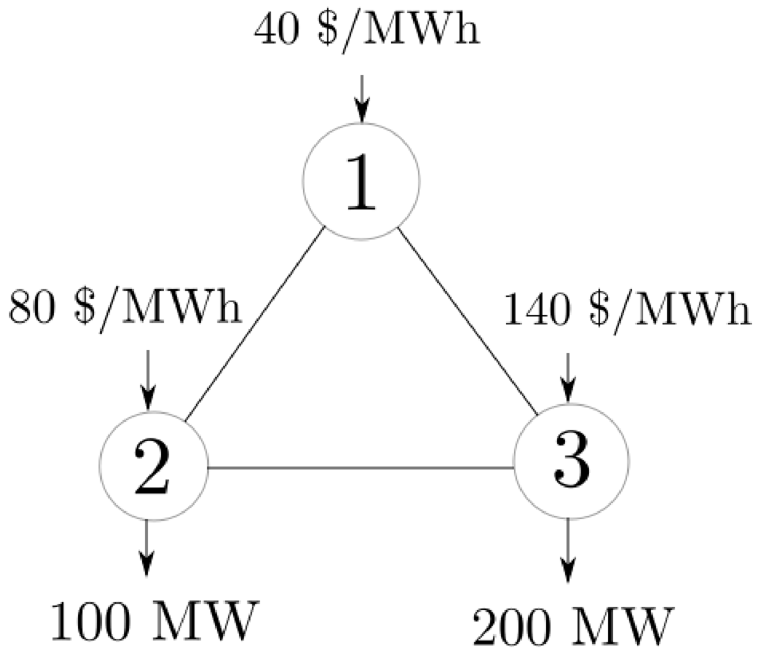

$\sum_{g \in G_m} p_g$
$\sum_{g \in G_n} p_g$
$f_{(m,n)} = \sum_{n' \in N} F_{(m,n),n'} r_{n'}$
r_m
r_n
$\sum_{l \in L_m} d_l$
$\sum_{g \in G_{\text{hub}}} p_g$
$\sum_{l \in L_n} d_l$
$r_{\text{hub}} = - \sum_{n' \neq \text{hub}} r_{n'}$
$\sum_{l \in L_{\text{hub}}} d_l$

Locational marginal pricing

Locational marginal pricing/nodal pricing: uniform price auction conducted as follows

- Sellers and buyers submit price-quantity pairs
- Market operator solves $(DCOPF)$ and announces ρ_n as market clearing price for bus n

Example



All lines have identical electrical characteristics (reactance)

Price splitting in neighboring nodes

Suppose $T_{1-2} = T_{2-3} = T_{1-3} = 50 \text{ MW}$

Lines 1-3, 2-3 should be used fully (can be proven graphically)

Optimal dispatch: $p_1 = 50 \text{ MW}, p_2 = 150 \text{ MW}, p_3 = 100 \text{ MW}$
Optimal flows: $f_{1-2} = 0 \text{ MW}, f_{2-3} = f_{1-3} = 50 \text{ MW}$

$\rho_1 = 40 \text{ \$/MWh}, \rho_2 = 80 \text{ \$/MWh}, \rho_3 = 140 \text{ \$/MWh}$ (why?)

Observe that $f_{1-2} < T_{1-2}$, but $\rho_2 > \rho_1$

Settlement of the LMP auction:

	Bid	Cleared	Payment ($/hour)
G1	+∞ MW at 40 $/MWh	50 MW at 40 $/MWh	2000
G2	+∞ MW at 80 $/MWh	150 MW at 80 $/MWh	12000
G3	+∞ MW at 140 $/MWh	100 MW at 140 $/MWh	14000
L2	100 MW at +∞ $/MWh	100 MW at 80 $/MWh	-8000
L3	200 MW at +∞ $/MWh	200 MW at 140 $/MWh	-28000

How much surplus is left over to the auctioneer?

LMP can be different from marginal costs

Suppose $T_{1-2} = 50 \text{ MW}, T_{2-3} = 100 \text{ MW}, T_{1-3} = 120 \text{ MW}$

Optimal dispatch: $p_1 = 160 \text{ MW}$, $p_2 = 140 \text{ MW}$, $p_3 = 0 \text{ MW}$
Optimal flows: $f_{1-2} = 40 \text{ MW}$, $f_{2-3} = 80 \text{ MW}, f_{1-3} = 120 \text{ MW}$

$\rho_3 = 120 \text{ \$/MWh}$ (use sensitivity)

Observe that ρ_3 is different from marginal cost of *all* generators

LMPs are not necessarily unique

Suppose $T_{1-2} = 50 \text{ MW}, T_{2-3} = 100 \text{ MW}, T_{1-3} = 100 \text{ MW}$

Optimal dispatch: $p_1 = 100 \text{ MW}$, $p_2 = 200 \text{ MW}$, $p_3 = 0 \text{ MW}$
Optimal flows: $f_{1-2} = 0 \text{ MW}$, $f_{2-3} = f_{1-3} = 100 \text{ MW}$

$\rho_3 = 140 \text{ \$/MWh}$ is a valid LMP (use sensitivity)
$\rho_3 = 120 \text{ \$/MWh}$ is a valid LMP (use sensitivity)

Observe that $120 \frac{\$}{\text{MWh}} \leq \rho_3 \leq 140 \frac{\$}{\text{MWh}}$ are all valid LMPs

Efficiency of LMP

If agents bid truthfully

1. Locational marginal pricing maximizes welfare, and
2. The resulting allocation maximizes the profit of agents *given* the market clearing price

Proof of item 1: LMP auction is solving welfare maximization problem

Proof of item 2: decomposition of KKT conditions of DCOPF

Producers

$$0 \leq p_g \perp MC_(p_g) - \rho_{n(g)} + \mu_g \geq 0$$
$$0 \leq \mu_g \perp P_g - p_g \geq 0$$
$$\Leftrightarrow$$
$$\max \rho_{n(g)} p_g - \int_0^{p_g} MC_g(x)dx$$
$$(\mu_g): \quad p_g \leq P_g$$
$$p_g \geq 0$$

Consumers

$$0 \leq d_l \perp -MB_l(d_l) + \rho_{n(l)} + \nu_l \geq 0$$
$$0 \leq \nu_l \perp D_l - d_l \geq 0$$
$$\Leftrightarrow$$
$$\max \int_0^{d_l} MB_l(x)dx - \rho_{n(l)} d_l$$
$$(\nu_l): d_l \leq D_l$$
$$d_l \geq 0$$

Transmission

$$\max_{f,r} \sum_{n \in N} \rho_n \cdot (-r_n)$$
$$(\psi_k): \quad f_k - \sum_{n \in N} F_{kn} \cdot r_n = 0, k \in K$$
$$(-\phi): \quad \sum_{n \in N} r_n = 0$$
$$(\lambda_k^+): \quad f_k \leq T_k, k \in K$$
$$(\lambda_k^-): \quad -f_k \leq T_k, k \in K$$

$$r_n - \sum_{g \in G_n} p_g + \sum_{l \in L_n} d_l = 0, n \in N$$

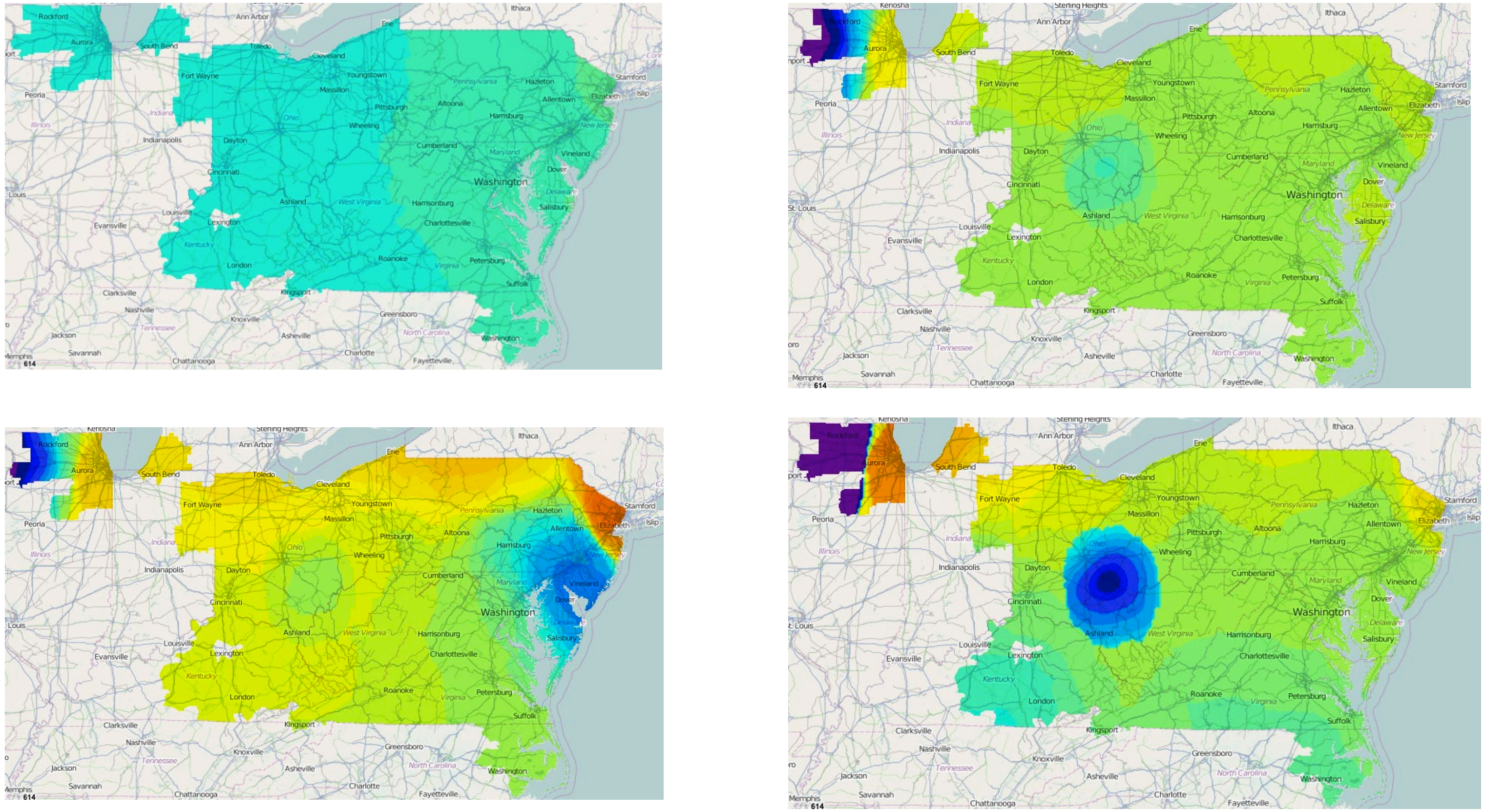
$$\Leftrightarrow$$

$$f_k - \sum_{n \in N} F_{kn} \cdot r_n = 0, k \in K$$
$$\sum_{n \in N} r_n = 0$$
$$\lambda_k^+ - \lambda_k^- + \psi_k = 0, k \in K$$
$$-\sum_{k \in K} F_{kn} \cdot \psi_k + \rho_n - \phi = 0, n \in N$$
$$0 \leq \lambda_k^+ \perp T_k - f_k \geq 0, k \in K$$
$$0 \leq \lambda_k^- \perp T_k + f_k \geq 0, k \in K$$

Market clearing

$$r_n - \sum_{g \in G_n} p_g + \sum_{l \in L_n} d_l = 0, n \in N$$

Nodal pricing in PJM (February 15, 2014)



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Upper right: 08:40

Lower left: 09:20

Lower right: 09:55

Congestion rent and congestion cost

Congestion rent and congestion cost

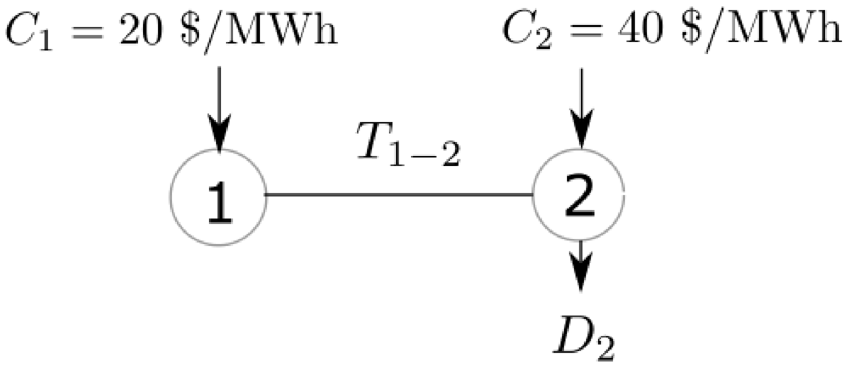
Congestion rent: surplus from locational price differences

$$CR = \sum_{n \in N} \rho_n \cdot (\sum_{l \in L_n} d_l - \sum_{g \in G_n} p_g)$$

Congestion cost: excess cost due to finite capacity of transmission lines

Congestion rent $\neq$ Congestion cost

Example: congestion rent ≥ congestion cost

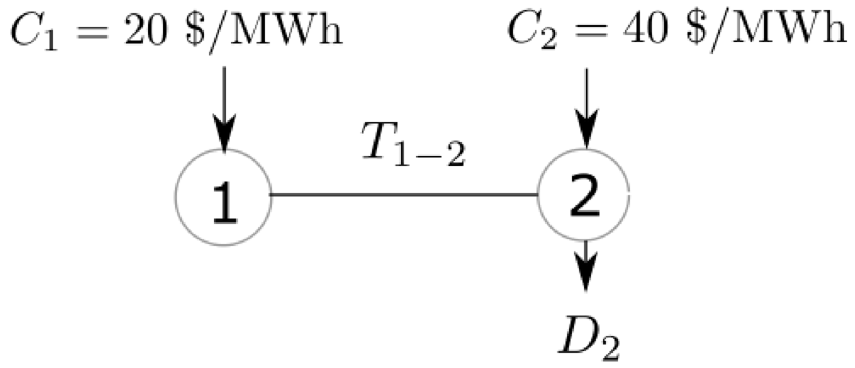


Suppose $D_2 = 50$ MW, $T_{1-2} = 50$ MW

Competitive market clearing prices: $\rho_1 = 20$ \$/MWh, $20 \frac{\$}{\text{MWh}} \leq \rho_2 \leq 40 \frac{\$}{\text{MWh}}$

Congestion rent: 0 – 1000 \$/h
Congestion cost: 0 \$/h

Example: congestion rent > congestion cost



Suppose $D_2 = 60\ \text{MW}$, $T_{1-2} = 50\ \text{MW}$

Market prices: $\rho_1 = 20 \frac{\$}{\text{MWh}}$, $\rho_2 = 40 \frac{\$}{\text{MWh}}$

Congestion rent: 1000 \$/h

Congestion cost: 200 \$/h

Congestion rent is non-negative

Congestion rent is non-negative, and given by the following expression:

$$CR = \sum_{n \in N} \rho_n \cdot \left(\sum_{l \in L_n} d_l - \sum_{g \in G_n} p_g\right) = \sum_{k \in K} (\lambda_k^+ + \lambda_k^-) \cdot T_k$$

Proof: If identity is true, then since $\lambda_k^+ \geq 0, \lambda_k^- \geq 0$, congestion rent is non-negative

$$\sum_{n\in N} \rho_n \cdot (\sum_{l\in L_n} d_l - \sum_{g\in G_n} p_g) =$$

Definition of r_n

$$-\sum_{n\in N} \rho_n \cdot r_n =$$

From $\rho_n = \sum_{k\in K} F_{kn} \cdot \psi_k + \varphi$ and $\psi_k = \lambda_k^- - \lambda_k^+$ and $\sum_{n\in N} r_n = 0$

$$\sum_{k\in K} (\lambda_k^+ - \lambda_k^-) \cdot \sum_{n\in N} F_{kn} \cdot r_n =$$

Definition of f_k

$$\sum_{k\in K} (\lambda_k^+ - \lambda_k^-) \cdot f_k =$$

Since $0 \leq \lambda_k^+ \perp T_k - f_k \geq 0$ and $0 \leq \lambda_k^- \perp T_k + f_k \geq 0$

$$\sum_{k\in K} (\lambda_k^+ + \lambda_k^-) \cdot T_k$$

Competitive market model for transmission capacity

Competitive market model with transmission

- Agents: power producers, power consumers
- Scarce resources (commodities): energy, transmission
- Profit maximization (quantity adjustment) of agents
- Market clearing (price adjustment) of commodities

- Assumption:
  - Producers responsible for shipping power *to* hub
  - Consumers responsible for shipping power *from* hub

Denote:

- φ: price of power
- λ_k^+/λ_k^-: price of transmission rights in/opposite to reference direction

Producer profit maximization:

$$\max_p \varphi \cdot p_g - \sum_{k \in K} \lambda_k^+ \cdot F_{k,n(g)} \cdot p_g + \sum_{k \in K} \lambda_k^- \cdot F_{k,n(g)} \cdot p_g - \int_0^{p_g} MC_g(x) dx$$

$$p_g \leq P_g$$

$$p_g \geq 0$$

Consumer surplus maximization:

$$\max_d \int_0^{d_l} MB_l(x)dx - \varphi \cdot d_l + \sum_{k\in K} \lambda_k^+ \cdot F_{k,n(l)} \cdot d_l - \sum_{k\in K} \lambda_k^- \cdot F_{k,n(l)} \cdot d_l$$
$$d_l \leq D_l$$
$$d_l \geq 0$$

Market clearing for energy:

$$\sum_{g\in G} p_g = \sum_{l\in L} d_l$$

Market clearing for transmission capacity:

$$0 \leq \lambda_k^+ \perp T_k - f_k \geq 0, k \in K$$
$$0 \leq \lambda_k^- \perp T_k + f_k \geq 0, k \in K$$

Efficiency of LMP

Nodal pricing produces an allocation of power and market clearing prices that correspond to a competitive market equilibrium. The converse is also true.

Proof: Compare KKT conditions of $(DCOPF)$ to KKT conditions of competitive market model

Losses

Losses as a function of injections

- We show in section B.6 that losses can be approximated as

$$lo = \sum_{k \in K} (L_{k0} + L_{k1} \cdot \sum_{n \in N} PTDF_{kn} \cdot r_n)$$

where

- $L_{k0} = -R_k \cdot \bar{P}_k^2, k \in K$
- $L_{k1} = 2 \cdot R_k \cdot \bar{P}_k, k \in K$
- $(\bar{P}_k, k \in K)$: vector of reference flows
- R_k: resistance of line k

Distribution of losses on nodes

- Denote the contribution of node n to losses as D_n
- Possible approach: contributions of nodes sum up to 1

$$\sum_{n \in N} D_n = 1$$

Optimal power flow model with losses

$$(DCOPF-L): \max_{p,d,r,r',f,lo} \sum_{l\in L} \int_0^{d_l} MB_l(x)\,dx - \sum_{g\in G} \int_0^{p_g} MC_g(x)dx$$

$$lo = \sum_{k\in K} (L_{k0} + L_{k1} \cdot \sum_{n\in N} PTDF_{kn} \cdot r_n)$$

$$-T_k \le f_k \le T_k, k \in K$$

$$f_k - \sum_{n\in N} F_{kn} \cdot r'_n = 0, k \in K$$

$$(\rho_n): r_n - \sum_{g\in G_n} p_g + \sum_{l\in L_n} d_l = 0, n \in N$$

$$r'_n = r_n - D_n \cdot lo, n \in N$$

$$\sum_{n\in N} r'_n = 0$$

$$p \ge 0, d \ge 0, lo \ge 0$$

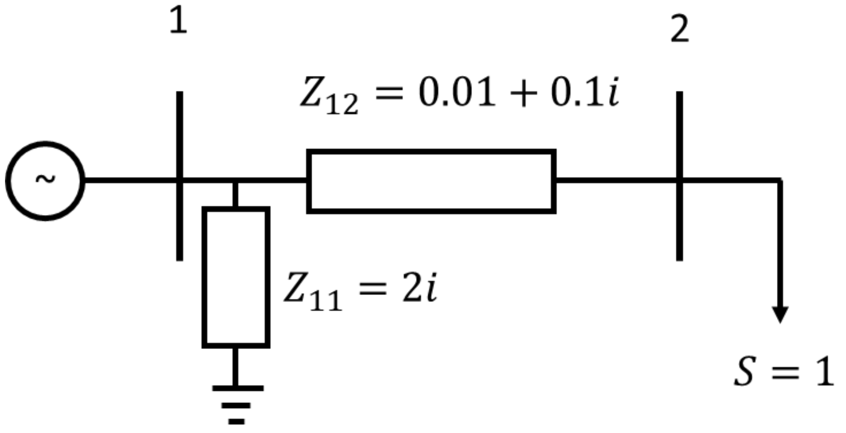
Observations

- ρ_n: market clearing price
- Price ρ_n now also accounts for losses
- r: net power injection before accounting for losses
- r': net power injection after accounting for losses

Example: locational marginal prices with losses on a two-node system

- Consider a two-node system with a generator in node 1 with marginal cost 20 $/MWh
- Suppose that
  - $D_1 = D_2 = 0.5$
  - $\bar{P}_{12} = 1$
- Thus

$$L_0 = -R_{12} \cdot \bar{P}_{12}^2 = -0.01$$
$$L_1 = 2 \cdot R_{12} \cdot \bar{P}_{12} = 0.02$$



Example: locational marginal prices

- Prices in model without losses: 20 $/MWh in both nodes
- Prices in model with losses:
  - 20 $/MWh in node 1
  - 20.41 $/MWh in node 2
- Economic interpretation: in order to get the power to node 2, one needs to pay the marginal cost of the power itself, but also the power lost in transmission
- Increase in losses: $2 \cdot R_{12} \cdot \bar{P}_{12}$
- Marginal cost of losses: $MC_{G_1} \cdot 2 \cdot R_{12} \cdot \bar{P}_{12} = 0.4$ $/MWh for $\bar{P}_{12} = 1$

References

[1] A. Papavasiliou, Optimization Models in Electricity Markets, Cambridge University Press

https://www.cambridge.org/highereducation/books/optimization-models-in-electricity-markets/0D2D36891FB5EB6AAC3A4EFC78A8F1D3#overview