Ancillary Services

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Source: chapter 6, Papavasiliou [1]

Outline

- Classification of ancillary services and reserves
- Co-optimization of energy and reserves
- Markets for reserves
  - Single type of reserve
  - Multiple types of reserve
- Operating reserve demand curves
- Balancing

Ancillary services

Ancillary services: services necessary to support the transmission of electric power from seller to purchaser given the obligations of control areas to maintain reliable operations

1. Scheduling and dispatch
2. Frequency containment reserve and frequency restoration reserve
3. Energy imbalance
4. Real power loss replacement
5. Voltage control
6. Load following

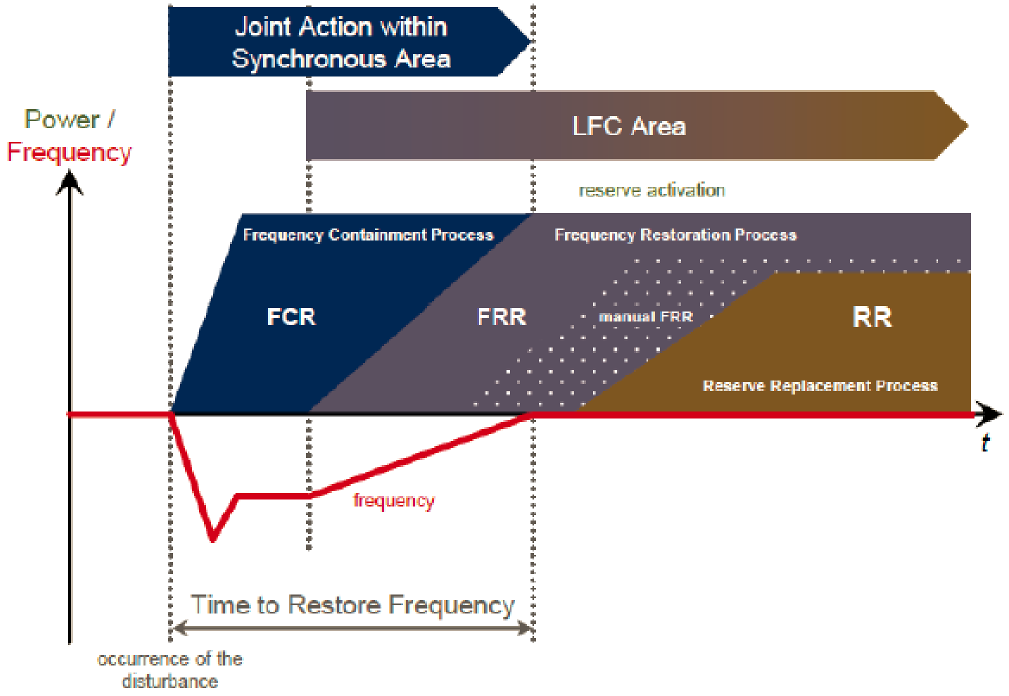
Classification of ancillary services and reserves

Uncertainty

- Continuous uncertainty: renewable energy and load forecast errors
- Discrete uncertainty/contingencies: outages of system components (transformers, transmission lines, generators, large loads)

Frequency containment and restoration

System frequency is an indicator of supply-demand balance



Frequency containment reserve

Frequency containment reserve (a.k.a. primary reserve, primary control) is the first line of defense

1. Change of inertia in generator rotors: immediate
2. Frequency-responsive governors (automatic controllers): reaction is immediate, may take a few seconds reach target
3. Automatic generation control (AGC, a.k.a. load frequency control, regulation): updated once every few seconds up to a minute

Automatic and manual frequency restoration reserve

Automatic and manual frequency restoration reserve (a.k.a. secondary reserve, frequency responsive reserve, secondary control, operating reserve): second line of defense

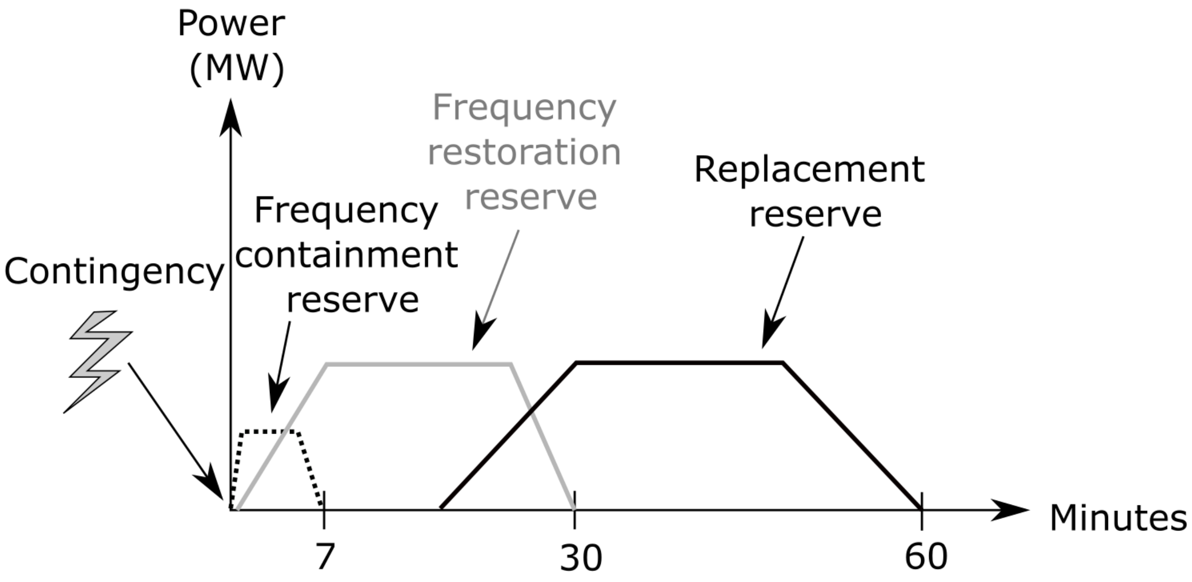
- Reaction in a few seconds, full response within 5-10 minutes
- Classified between spinning and non-spinning reserve
  - **Spinning reserve**: generators that are on-line
  - **Non-spinning reserve:** generators that are off-line but can start rapidly
- Requirements dictated by capacity of greatest generator in the system and forecast errors

Replacement reserve

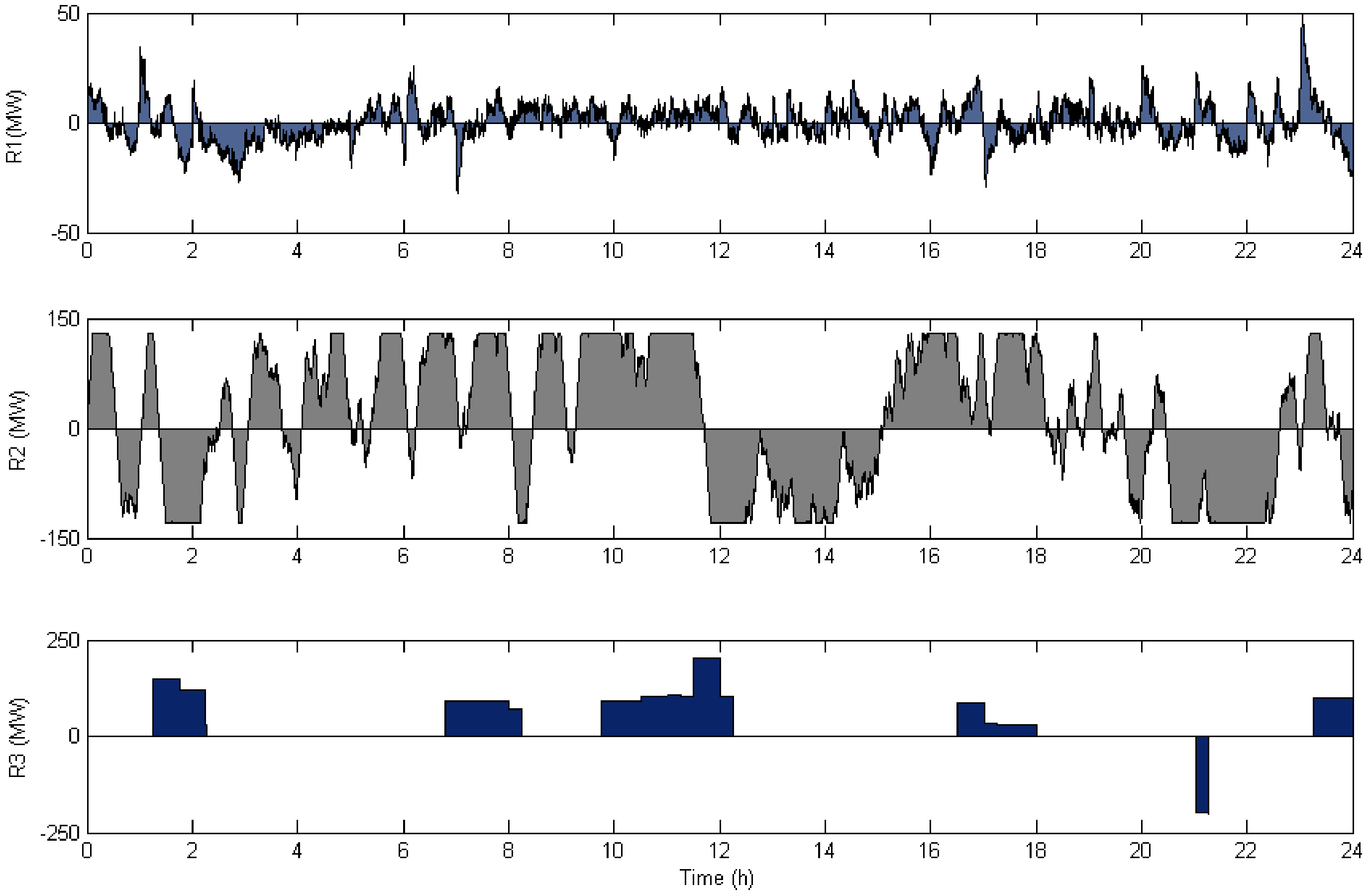
Replacement reserve (a.k.a. tertiary control, tertiary reserve, replacement reserve): third line of defense

- Available within a few (e.g. 15) minutes

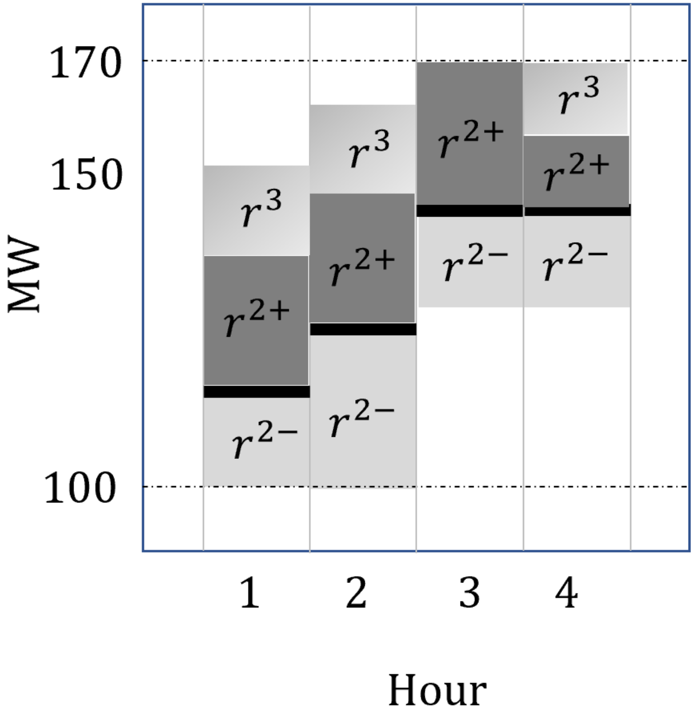
Sequential activation of reserves



Reserves in Belgium



Example 6.1: frequency restoration reserves and replacement reserves

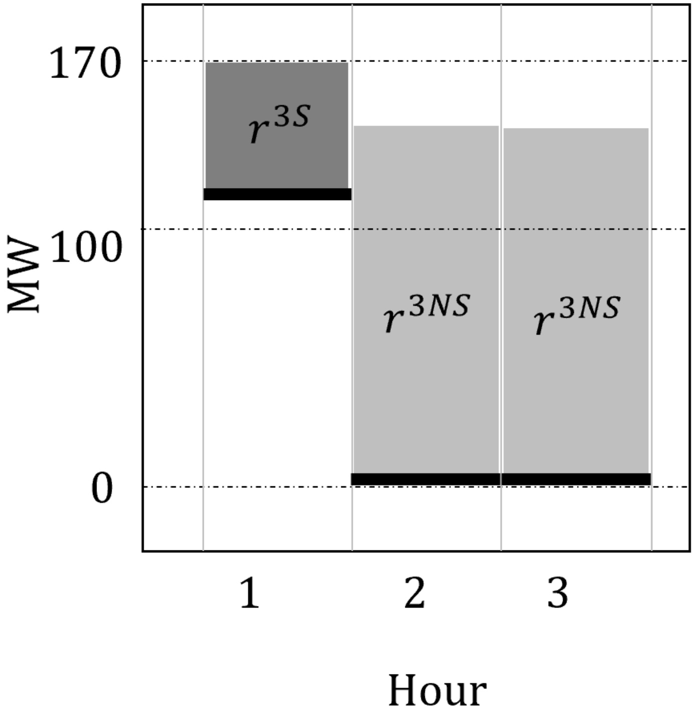


Suppose:

- Upward/downward frequency restoration reserve limit: 20 MW
- Replacement reserve limit: 10 MW
- Min capacity: 100 MW
- Max capacity: 170 MW
- Planned production: 110 MW (hour 1), 120 MW (hour 2), 150 MW (hour 3), 150 MW (hour 4)

- How much downward restoration reserve?
- How much upward restoration reserve in hours 1, 2? In hours 3, 4?
- How much replacement reserve in hours 1, 2? In hours 3, 4?

Example 6.2: interaction of spinning and non-spinning reserve



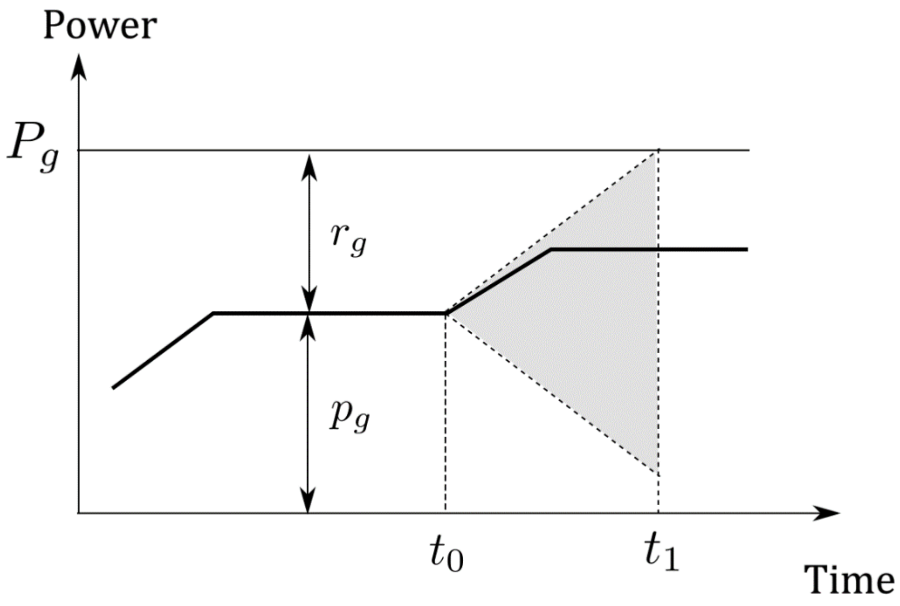
Suppose:

- Non-spin reserve limit: 150 MW
- Min capacity: 100 MW
- Max capacity: 170 MW
- Planned production: 110 MW (hour 1), 0 MW (hour 2), 0 MW (hour 3)

How much spinning reserve in hour 1? How much non-spinning reserve in hours 2, 3?

Co-optimization of energy and reserves

Modeling reserve constraints



Gray indicates ramp rate, r_g can be offered as reserve at t_0 if response time is at least $t_1 - t_0$

Factors that limit amount of available reserve r_g:

- Generator capacity P_g

$$p_g + r_g \leq P_g$$

- Generator ramp rate R_g

$$r_g \leq R_g$$

- Note: R_g depends on *type* (containment reserve, restoration reserve, replacement reserve) of offered reserve

- Denote R as total reserve requirement:

$$\sum_{g \in G} r_g \geq R$$

Co-optimization of energy and reserve

Assume:

- No transmission constraints
- Single type of reserve

$$(EDR):\ \max_{p,d,r} \int_0^D MB(x)dx - \sum_{g\in G} \int_0^{p_g} MC_g(x)dx$$

$$(\lambda): d - \sum_{g\in G} p_g = 0$$

$$(\mu): R \leq \sum_{g\in G} r_g$$

$$r_g \leq R_g, g \in G$$

$$p_g + r_g \leq P_g, g \in G$$

$$p_g \geq 0, r_g \geq 0, g \in G$$

$$d \geq 0$$

Example 6.3: provision of reserve by the most expensive units

- Full activation time: 10 minutes
- Three generators
- Inelastic demand $D = 100$ MW
- Replacement reserve requirement $R = 100$ MW (why 100?)

Generator	Marginal cost ($/MWh)	Max (MW)	Ramp (MW/minute)
Cheap	0	100	$+\infty$
Moderately expensive	10	100	1
Expensive	80	100	5

Optimal solution: use most expensive generators for providing reserve

Solve for reserve first, in order of decreasing marginal cost:

- $r_2 = 50$ MW
- $r_1 = 10$ MW
- $r_3 = 40$ MW

Then, solve for energy, in order of increasing marginal cost:

- $p_3 = 60$ MW
- $p_1 = 40$ MW
- $p_2 = 0$ MW

Additional features

Notation

- $R1^+$, $R1^-$: upward/downward frequency containment reserve requirement
- $R2$, $R3$: restoration and replacement reserve requirement
- $r1^+_{g,1}$, $r1^+_{g,2}$, $r1^+_{g,3}$: fast capacity allocated to containment/restoration/replacement reserve
- $r1^-_g$: fast capacity allocated to downward containment reserve
- $r2_{g,2}/r2_{g,3}$: moderately fast capacity allocated to restoration/replacement reserve
- $r3_g$: slow capacity allocated to replacement reserve
- $R1_g$, $R2_g$, $R3_g$: amount of frequency containment/frequency restoration/frequency replacement reserve that a unit can make available

- One-way substitutability: frequency containment reserve $\succ$ frequency restoration reserve $\succ$ replacement reserve:

$$\sum_{g\in G} r1_{g,1}^{+} \geq R1^{+}, \sum_{g\in G} r1_{g}^{-} \geq R1^{-},$$

$$\sum_{g\in G} (r1_{g,2}^{+} + r2_{g,2}) \geq R2, \sum_{g\in G} \left(r1_{g,3}^{+} + r2_{g,3} + r3_{g}\right) \geq R3$$

- Technical min and max:

$$p_g + \sum_{i=1}^{3} r1_{g,i}^{+} + \sum_{i=2}^{3} r2_{g,i} + r3_g \leq P_g, p_g - r1_g^{-} \geq 0, g \in G$$

- Ramp constraints:

$$\sum_{i=1}^{3} r1_{g,i}^{+} \leq R1_g, r1_g^{-} \leq R1_g, \sum_{i=2}^{3} r2_{g,i} \leq R2_g, r3_g \leq R3_g, g \in G$$

Security constrained economic dispatch (SCED)

SCED: two-stage model that determines secondary reserve by representing contingencies *within* the model

- ω: contingency
- p_g: first-stage decisions
- $p_g(\omega)$: second-stage decisions
- Constraint linking first and second stage:

$$-R_g \leq p_g(\omega) - p_g \leq R_g$$

$$(SCED): \min_p \sum_{g \in G} \int_0^{p_g} MC_g(x)dx$$

$$p_g \leq P_g, g \in G$$

$$\sum_{g \in G} p_g = D$$

$$p_g(\omega) \leq P_g \cdot 1_g(\omega), g \in G, \omega \in \Omega$$

$$\sum_{g \in G} p_g\,(\omega) = D$$

$$-R_g \leq p_g(\omega) - p_g \leq R_g, g \in G, \text{αν}\ 1_g(\omega) = 1$$

$$p_g \geq 0, g \in G$$

$$p_g(\omega), g \in G, \omega \in \Omega$$

Note:

- Demand is inelastic, *not* a decision ⇒ all demand must be satisfied for all ω
- Objective function: cost of the base case (no contingencies)

- D: system demand
- If $1_g(\omega) = 0$, then generator g is not available in contingency ω
- $\boldsymbol{N-1}$ **security**: being able to serve demand with $N-1$ components (i.e. outage of one component)
- $\boldsymbol{N-k}$ **security**: being able to serve demand with $N-k$ components (i.e. outage of k components)

How do we model *N-1* security using $(SCED)$?

Which model is easier to solve, (EDR) or $(SCED)$?

Example 6.4: security constrained economic dispatch

- Three generators
- Inelastic demand $D = 100$ MW

- The $(SCED)$ solution is identical to the (EDR) solution: $p_1 = 40$ MW, $p_2 = 0$ MW, $p_3 = 60$ MW

Generator	Marginal cost ($/MWh)	Max (MW)	Ramp (MW/min)
Cheap	0	100	$+\infty$
Moderately expensive	10	100	1
Expensive	80	100	5

… but the solution could have been different if (EDR) had a different reserve requirement R

What is the response when generator 2 is unavailable?

Import constraints

Import constraints limit total power flow on sensitive groups of lines, and protect against unplanned outages

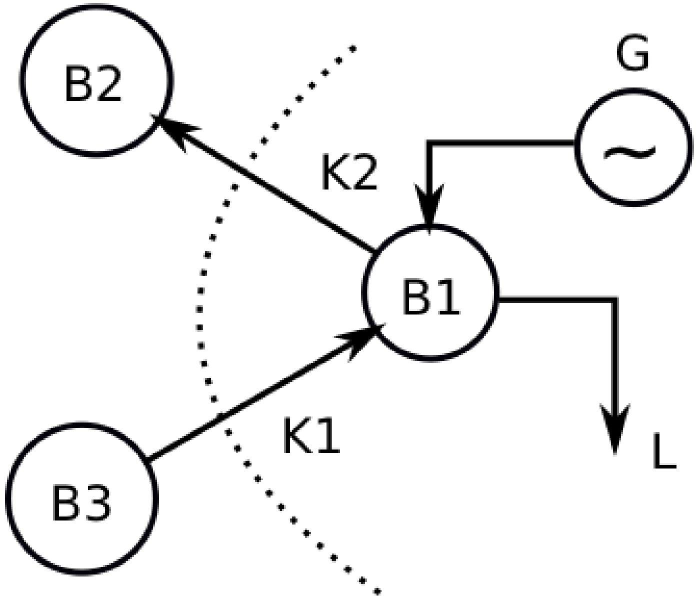
$$\sum_{k \in IG_j} \gamma_{jk} \cdot f_k \leq IC_j, j \in IG$$

- IG: set of import groups
- γ_{jk}: reference direction
- IG_j: set of lines in import group j
- IC_j: flow limit over import group
- f_k: flow on line k

Example 6.5: import constraints

Logic: if generator G within load pocket $B1$ fails, power needs to come from outside

$$f_{K1} - f_{K2} \leq 100 \text{ MW}$$



- $IG = \{IG_1\}$
- $IC_{IG_1} = 100$ MW
- $\gamma_{IG_1,K_1} = 1, \gamma_{IG_1,K_2} = -1$

Markets for reserve

Single type of reserve

Multiple types of reserve

Simultaneous auction for energy and reserve

Coordination constraints of (EDR):

- Supply equals demand:

$$d - \sum_{g \in G} p_g = 0$$

- Reserve requirements:

$$\sum_{g \in G} r_g \geq R$$

Simultaneous auction for energy and reserve:

- Suppliers submit *ramp rates* and increasing bids. Buyers submit decreasing bids.
- Market operator solves (EDR) and announces λ as market clearing price for power, μ as market clearing price for reserve

- Note: generators submit ramp rates as part of bid
- Power bought by loads from generators
- Reserve bought by market operator from generators

Example 6.6: co-optimization prices induce the optimal dispatch

- Three generators
- Inelastic demand $D = 100$ MW
- Frequency restoration reserve requirement (response in 10 minutes): $R = 100$ MW

Generator	Marginal cost ($/MWh)	PMax (MW)	Ramp rate limit (MW/min)
Cheap	0	100	$+\infty$
Moderately expensive	10	100	1
Expensive	80	100	5

Prices:

- Energy: $\lambda^* = 10$ $/MWh
- Reserve: $\mu^* = 10$ $/MWh

Transfers:

- Loads pay generators $1000 per hour for energy
- System operator pays generators $1000 per hour for reserve

- Generator 1
    - Reserve market offers profit of 10 \$/MWh, energy market offers profit of 0 \$/MWh
    - Profit-maximizing reserve: 10 MW
    - Profit-maximizing energy: indifferent
- Generator 2
    - Reserve market offers profit of 10 \$/MWh, energy market offers profit of -70 \$/MWh
    - Profit-maximizing reserve: 50 MW
    - Profit-maximizing energy: 0 MW
- Generator 3
    - Reserve market offers profit of 10 \$/MWh, energy market offers profit of 10 \$/MWh
    - Profit-maximizing energy + reserve: 100 MW

Sequential markets for reserve and energy

In markets without co-optimization, we often have the following auctions, one after the other:

- First step: reserve auction
- Second step: energy auction

Example 6.7: sequential clearing requires anticipation of prices

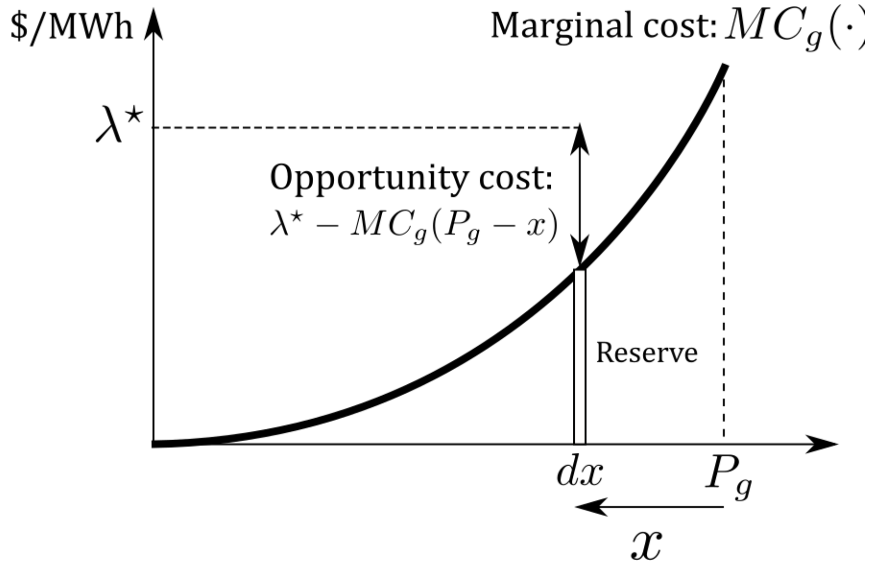
- Three generators
- Inelastic demand $D = 100$ MW
- Frequency restoration reserve requirement: $R = 100$ MW

Generator	Marginal cost ($/MWh)	PMax (MW)	Ramp rate limit (MW/min)
Cheap	0	100	$+\infty$
Moderately expensive	10	100	1
Expensive	80	100	5

- Suppose all agents believe the energy price will be λ^* and bid truthfully, generator g bids **opportunity cost**:

$$\max(\lambda^* - MC_g, 0)$$

Opportunity cost



Allocate slice dx for reserves, instead of using it to sell energy at a price λ^* ⇒ **opportunity cost**:

$$\max(0, \lambda^* - MC_g(p_g - x))$$

Uniform price auction for reserve:

- Generator 1 cleared for 40 MW
- Generator 2 cleared for 10 MW
- Generator 3 cleared for 50 MW

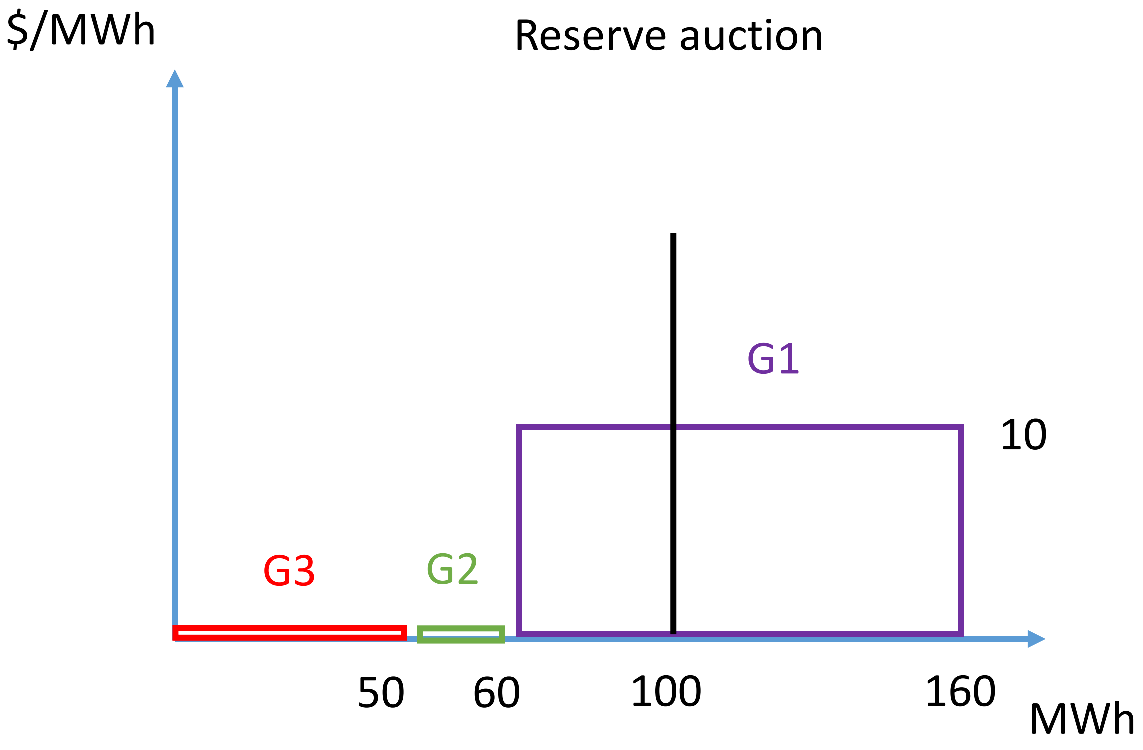
Uniform price auction for energy:

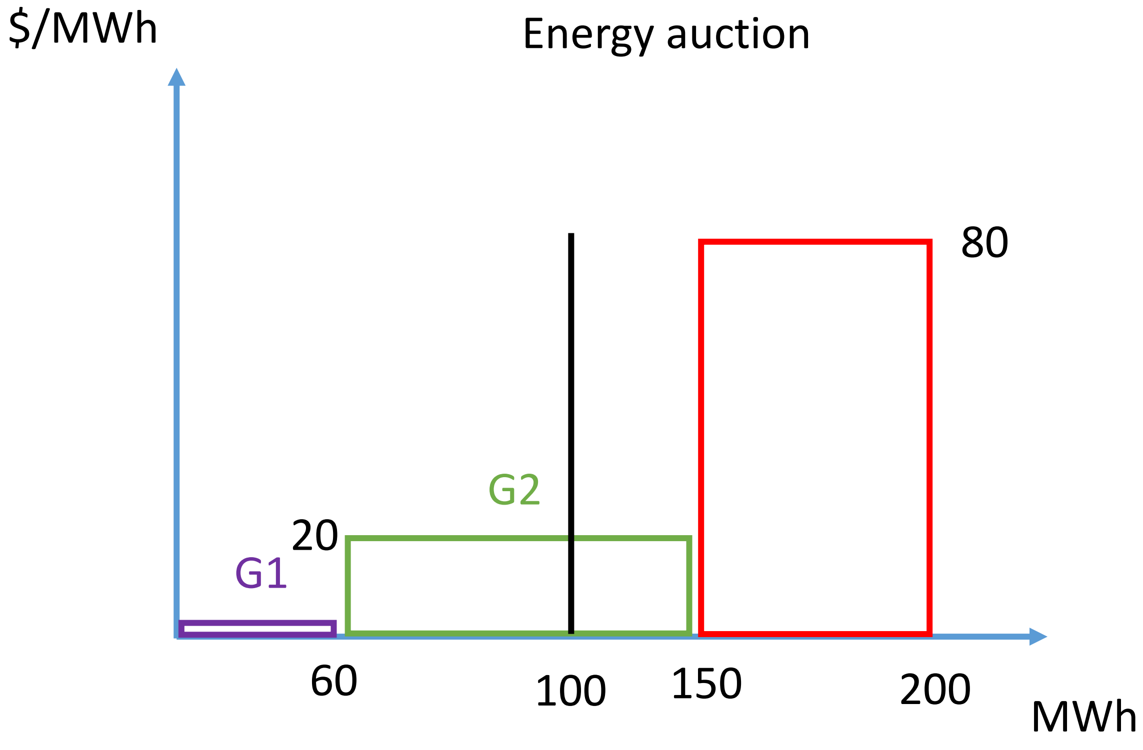
- Generator 1: offers 60 MW at 0 $/MWh
- Generator 2: offers 90 MW at 20 $/MWh
- Generator 3: offers 50 MW at 80 $/MWh

Energy market clearing price: λ^* = 10 $/MWh

Returning to reserve auction, we find that μ^* = 10 €/MWh

Sequential clearing of reserve and energy





Markets for reserve

Single type of reserve

Multiple types of reserve

Market design for reserve auctions

- We saw that sequential clearing of reserves and energy is equivalent to simultaneous clearing
  - Should the auctions be pay-as-bid or uniform price?
  - Should the auctions for different reserves be simultaneous or sequential?

Complicating factor: one-way substitutability

Frequency containment reserve $\succ$ frequency restoration reserve $\succ$ replacement reserve

Example 6.8: price reversals

- Demand for frequency containment reserve: 400 MW
- Demand for frequency restoration reserve: 350 MW
- Bid 1: 600 MW for frequency containment reserve at 10 $/MWh
- Bid 2: 50 MW for frequency containment reserve at 15 $/MWh
- Bid 3: 25 MW for frequency restoration reserve at 5 $/MWh
- Bid 4: 400 MW for frequency restoration reserve at 20 $/MWh

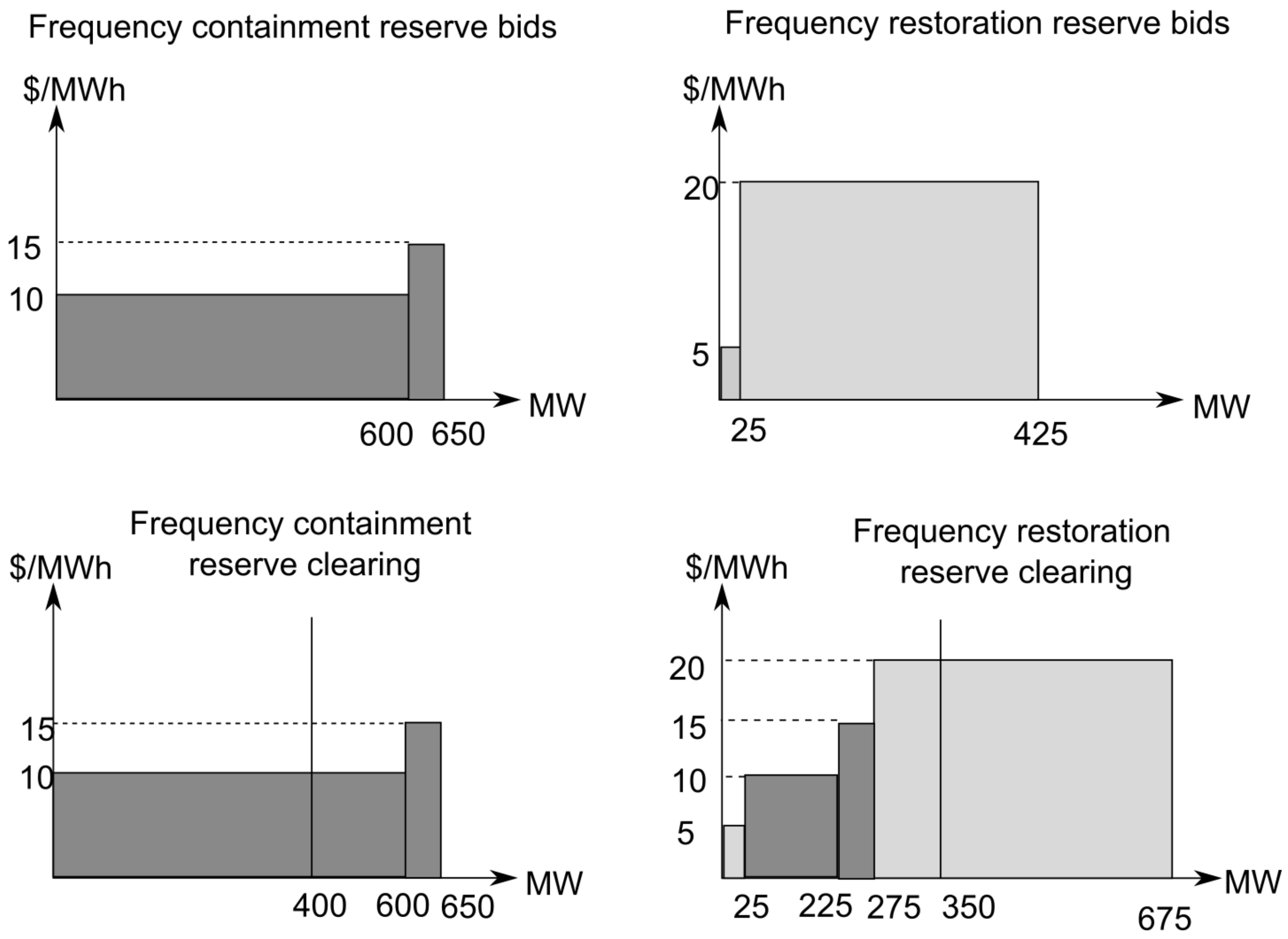
We consider three auction designs:

- Cascading 1
- Cascading 2
- Simultaneous clearing

Example 6.8: cascading design #1

- Clearing of frequency containment reserve → cascade of leftover bids → clearing of frequency restoration reserve
- Uniform price based on most expensive accepted bid in current auction

- Price of frequency containment reserve: 10 $/MWh
- Price of frequency restoration reserve: 20 $/MWh
- Price reversals (this is bad)
- Cost: $8375
- Payment: $11000


Frequency containment reserve bids
$/MWh
15
10
MW
600 650
Frequency restoration reserve bids
$/MWh
20
5
MW
25
425
Frequency containment reserve clearing
$/MWh
15
10
MW
400 600 650
Frequency restoration reserve clearing
$/MWh
20
15
10
5
MW
25 225 275 350
675

Example 6.9: cascading design #2

- Clearing of frequency containment reserve → cascade of leftover bids → clearing of frequency restoration reserve
- Uniform price based on most expensive accepted bid in current auction or auctions of lower quality

- Price of frequency containment reserve: 15 $/MWh
- Price of frequency restoration reserve: 20 $/MWh
- Price reversals
- Cost: $8375
- Payment: $13000

Simultaneous clearing

$$(Res):\ \min_{r1,r2} \sum_{g \in G} \int_0^{r1_{g,1}+r1_{g,2}+r2_g} OC_g(x)dx$$

$$(\mu 1): \sum_{g \in G} r1_{g,1} \geq R1$$

$$(\mu 2): \sum_{g \in G} (r1_{g,2} + r2_g) \geq R2$$

$$(\rho 1_g): r1_{g,1} + r1_{g,2} \leq R1_g, g \in G$$

$$(\rho 2_g): r2_g \leq R2_g, g \in G$$

$$r1_{g,1} \geq 0, r1_{g,2} \geq 0, r2_g \geq 0, g \in G$$

A simultaneous uniform pricing auction for reserves is conducted as follows:

- Suppliers submit incremental bids for reserves: price-quantity pairs that indicate the amount of reserves that they are willing to provide for a given price
- The market operator solves (Res) and announces $\mu 1$ as the uniform price for frequency containment reserve, and $\mu 2$ as the price for frequency restoration reserve

Preventing price reversals

In the simultaneous uniform price auction the price for higher quality reserve is higher: $\mu 1 \geq \mu 2$

Proof

- KKT conditions:

$$0 \leq r1_{g,1} \perp MC_g\left(r1_{g,1} + r1_{g,2} + r2_g\right) - \mu 1 + \rho 1_g \geq 0, g \in G$$
$$0 \leq r1_{g,2} \perp MC_g\left(r1_{g,1} + r1_{g,2} + r2_g\right) - \mu 2 + \rho 1_g \geq 0, g \in G$$

- Since $R1 > 0$, it must be the case that $r1_{g,1} > 0$ for some g

$$\mu 1 = MC_g\left(r1_{g,1} + r1_{g,2} + r2_g\right) + \rho 1_g$$

- The conclusion follows since

$$\mu 2 \leq MC_g\left(r1_{g,1} + r1_{g,2} + r2_g\right) + \rho 1_g$$

Example 6.10: correction of price reversals

- Price of frequency containment reserve: 20 $/MWh
- Price of frequency restoration reserve: 20 $/MWh
- Cost: $8375
- Payment: $15000

Criticism: high payments to generators, in order to induce them to bid truthfully

Operating reserve demand curves

Price variability in scarcity conditions

- A drawback of markets with inelastic energy demand is that prices can be highly volatile
- Specifically, in scarcity conditions:
  - If the system is on the verge of load shedding, the market price can be the marginal cost of the marginal unit (e.g. 150 $/MWh)
  - While if there is load shedding the price shoots to VOLL (e.g. 10000 $/MWh)

Example 6.11: price volatility

- D MW of inelastic demand
- VOLL: 1000 $/MWh
- 100 MW of elastic demand
- Thus:

$$MB_L(x) = \begin{cases} 1000 \frac{\$}{\text{MWh}}, 0 \text{ MW} \leq x \leq D \text{ MW} \\ 1000 - 10 \cdot (x - D) \frac{\$}{\text{MWh}}, D \text{ MW} < x < D + 100 \text{ MW} \end{cases}$$

- Marginal cost curve:

$$MC_G(x) = 0.015 \cdot x \frac{\$}{\text{MWh}}$$

Example 6.11: prices with inelastic reserve requirement

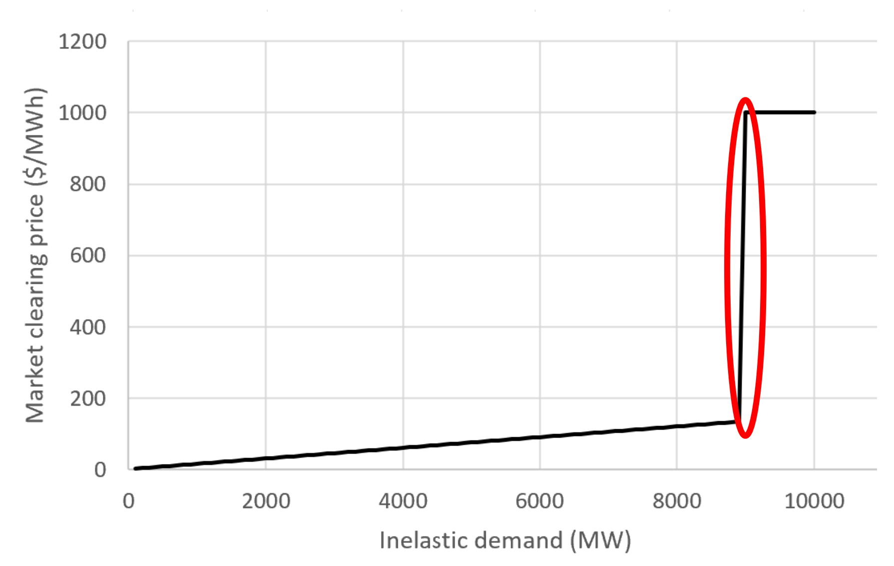
- Suppose an inelastic reserve requirement of $R = 1000$ MW
- It can be shown that the market price behaves as follows:

$$\lambda^* = \begin{cases} 0.015 \cdot (0.9985 \cdot D + 99.85)\,\frac{\$}{\text{MWh}}, 0\text{ MW} \leq D \leq 8913.5\text{ MW} \\ 1000 - 10 \cdot (9000 - D)\,\frac{\$}{\text{MWh}}, 8913.5\text{ MW} < D \leq 9000\text{ MW} \\ 1000\,\frac{\$}{\text{MWh}}, D > 9000\text{ MW} \end{cases}$$

- Γιατί; Η ανελαστική εφεδρεία ισοδυναμεί με το να θέσουμε την απαίτηση εφεδρείας σε μια αποτίμηση μεγαλύτερη από τη μέγιστη αποτίμηση της συνάρτησης ζήτησης

Example 6.11: prices with inelastic reserve requirement

- Price changes abruptly:
  - 135 \$/MWh at 8913.5 MW of demand
  - 1000 \$/MWh at 9000 MW of demand
- Price volatility ⇒ investment risk (-)



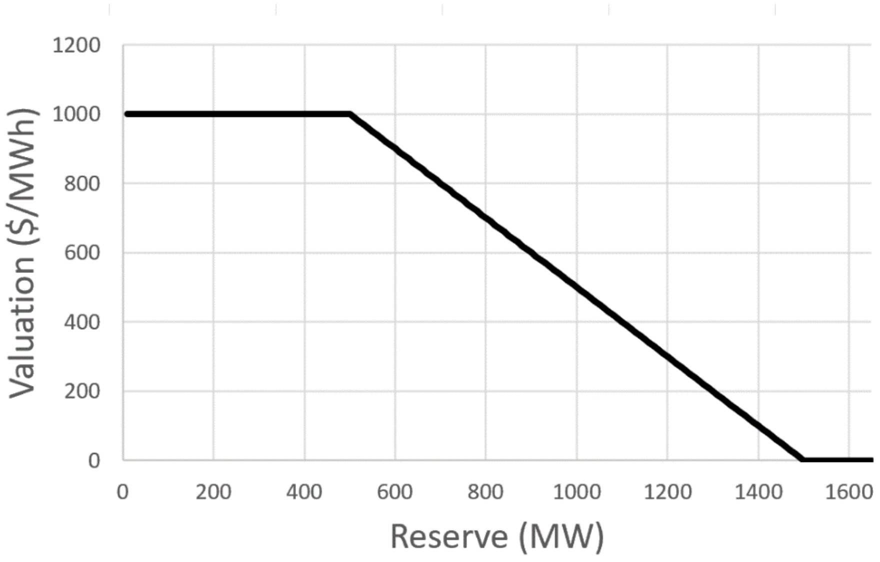
Operating reserve demand curves

- **Operating reserve demand curves (ORDCs)**: measure for mitigating high volatility in market prices with limited demand elasticity
- Idea: introduce elasticity in the demand for reserve

Operating reserve demand curve

Intuition:

- For reserve below R_m the system operator is willing to pay a high price, in order to avoid system collapse
- For reserve above R_M the system operator is not willing to pay anything, because the system is already secure



Co-optimization of energy and reserve with ORDCs

- Denote $MR(x)$ the marginal benefit for available reserve
- Market model:

$$(ORDC): \max_{p,d,r,dr} \int_0^d MB(x)dx + \int_0^{dr} MR(x)dx - \sum_{g \in G} \int_0^{p_g} MC_g(x)dx$$

$$(\lambda): d - \sum_{g \in G} p_g = 0$$

$$(\mu): dr - \sum_{g \in G} r_g = 0$$

$$r_g \leq R_g, g \in G$$
$$p_g + r_g \leq P_g, g \in G$$

$$p, d, r, dr \geq 0$$

Auctions with ORDCs

A uniform price auction based on an ORDC is conducted as follows:

- Producers submit increasing bids for energy, consumers submit decreasing bids for energy
- The system operator submits decreasing offers for reserve
- The market operator solves $(ORDC)$ and announces λ as the energy price, and μ as the reserve price

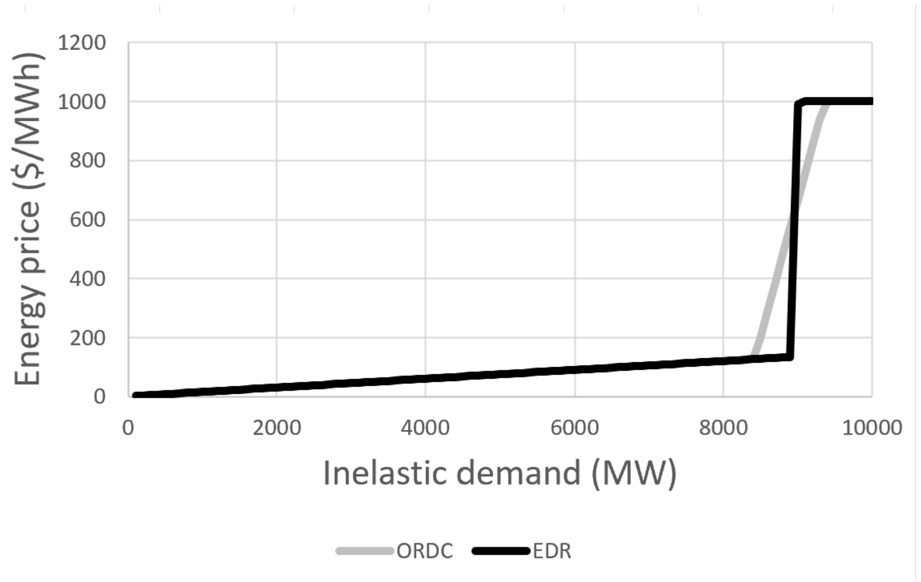
Reducing energy price volatility through ORDCs

The energy price behaves more smoothly with ORDCs:

- A peaking unit g that splits its capacity between energy and reserve must earn an equal profit margin: $\mu = \lambda - MC_g$
- Thus energy price λ and reserve price μ differ only by the marginal cost of the marginal unit
- And due to the elasticity of the ORDC reserve prices behave smoothly
- This "anchors" energy prices, which also behave smoothly
- And this despite energy demand being inelastic!

Example 6.12: reducing energy price volatility through an ORDCs

- Suppose that we replace the inelastic reserve requirement for 1000 MW with an ORDC
- ORDC parameters:
  - $R_m = 500$ MW
  - $R_M = 1500$ MW
  - $VR_m = 1000$ \$/MWh
- Note that the prices behave more smoothly as a function of demand



Shape of ORDC

- The shape of the ORDC determines how reserve prices behave
- Alternative shapes:
  - Inelastic curves: corresponds to existing inelastic reserve requirements which show up in many systems
  - ORDCs with steps: Ireland, ISO-NE, MISO, CAISO, SPP
  - ORDCs depending on VOLL and loss of load probability (LOLP): used or considered in a number of systems (PJM, ERCOT, Belgium, UK, Greece, Poland)

ORDC based on VOLL and LOLP

- Proposal for ORDC depending on VOLL and LOLP [2]:

$$MR(x) = \left(VOLL - \widehat{MC}\right) \cdot LOLP(x)$$

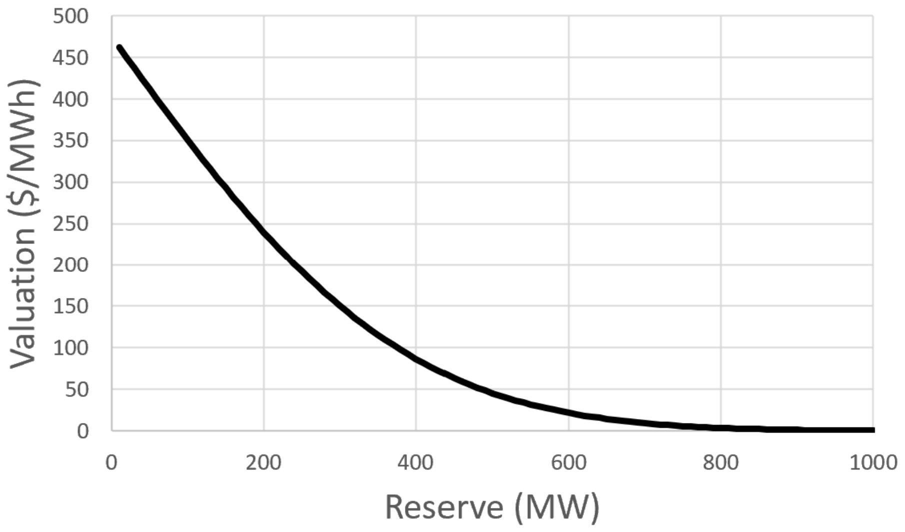
- Where:
  - VOLL: value of lost load
  - $\widehat{MC}$: approximation of marginal cost for producing additional energy
  - $LOLP(x)$: loss of load probability given that the system has x MW of reserve
- Intuition: the incremental value of an additional MW of reserve is proportional to the contribution of that MW in limiting the probability of loss of load

Example 6.13: ORDC based on VOLL and LOLP

- Consider a system with
  - VOLL equal to 1000 \$/MWh
  - Marginal cost $\widehat{MC} = 50$ \$/MWh
  - Normal distribution of imbalances with a mean value of 0 MW and standard deviation of 300 MW
- The ORDC is computed as follows:

$$MR(x) = 950 \cdot \left(1 - \Phi_{0,300}(x)\right)$$

- Here, $\Phi_{\mu,\sigma}(\cdot)$ is the cumulative distribution function of a normal distribution with mean μ and standard deviation σ



Certain virtues of ORDCs

- Flexible producers are paid for helping the system at the moment when the system needs them most: **pay for performance**
- Flexible producers are paid not only for energy but also for the availability of reserve
- The mechanism is implemented in real-time markets, but through back-propagation (chapter 9) it creates a robust investment signal in forward reserve markets

Example 6.14: remuneration of reserves in real time

- Suppose that the (ORDC) model produces the following prices:
  - Energy price: $\lambda^* = 60$ \$/MWh
  - Reserve price: $\mu^* = 10$ \$/MWh
- And let us assume that:
  - The unit has a technical maximum of 100 MW
  - The unit produces 10 MW of energy in real time
  - The unit has not sold energy/reserve in forward markets
- Payments:
  - Energy: $60 \frac{\$}{\mathrm{MWh}} \times 10 \text{ MWh} = \$600$
  - Reserve: $10 \frac{\$}{\mathrm{MWh}} \times 90 \text{ MWh} = \$900$

ORDC/scarcity adders

- Certain markets (e.g. the European) only solve real-time economic dispatch (and not the $(ORDC)$ model of slide 58)
- This does not mean that we cannot implement the mechanism of slide 58 (e.g. Texas)
- How? With the ex post computation of **ORDC adders/scarcity adders**

Example 6.15: ORDC adders

- Consider a system without energy and reserves co-optimization
- Real-time energy price: $\widehat{MC} = 50 \frac{\$}{\text{MWh}}$
- Available real-time reserve: 600 MW (measured through telemetry)
- The unit that we are interested in produces 10 MW and has a capacity of 100 MW
- Scarcity adder based on the equation of slide 63:

$$\tilde{\mu} = \left(VOLL - \widehat{MC}\right) \cdot LOLP(R) = (1000 - 50) \cdot \left(1 - \Phi_{0,300}(600)\right)$$
$$= 21.61 \frac{\$}{\text{MWh}}$$

Comparing payments in example 6.15

- Compensation in an energy-only market, without ORDC adder:
  $50 \frac{\$}{\mathrm{MWh}} \times 10 \text{ MWh} = \$500$
- Compensation in a market that trades reserve, with an ORDC adder:
  - Energy payments: $(50 + 21.61) \frac{\$}{\mathrm{MWh}} \times 10 \text{ MWh} = \$716.1$
  - Reserve payments: $21.61 \frac{\$}{\mathrm{MWh}} \times 90 \text{ MWh} = \$1944.9$
- The overall effect: relative to an energy-only market, the unit is essentially compensated by the ORDC adder ($21.61 \frac{\$}{\mathrm{MWh}}$) for 100 MW of capacity that it makes available in real time, whether these MW are used as energy or reserve

Balancing

What is balancing?

Balancing is the task of *adjusting* power production and consumption in *real time*

What does this have to do with reserve? Balancing is offered by

- **Balancing service providers (BSPs)**: resources that have committed to offer *reserve*. Reserves are obliged to offer an amount of power at least equal to the amount of their promised reserve capacity
- **Free bids**: resources that offer balancing energy without being obliged to do so

Increment/decrement bids

To run a balancing market using increment/decrement bids:

- Collect bids by resources that can adjust their production or consumption in real time
- Activate these resources in order to relieve any imbalances
- Charge market participants who deviate from their earlier positions

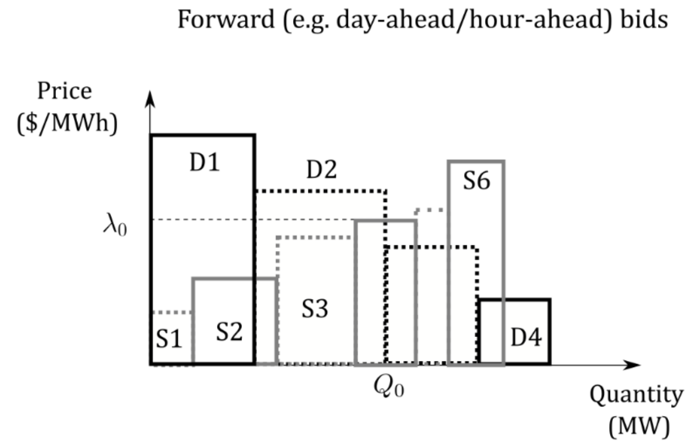
Logic of increment/decrement bids

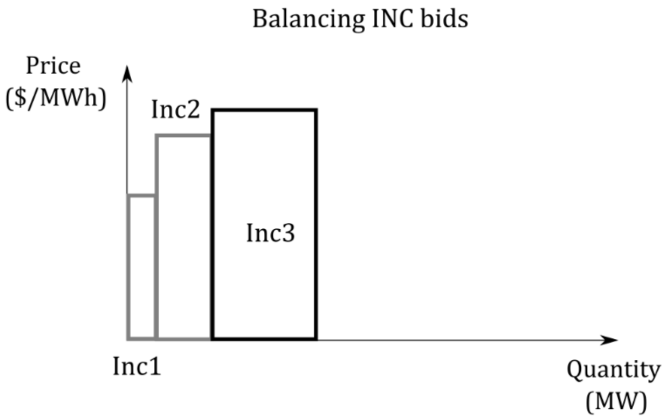
Suppose that a resource has been cleared for Q_0 MW at P_0 $/MWh in the day-ahead market

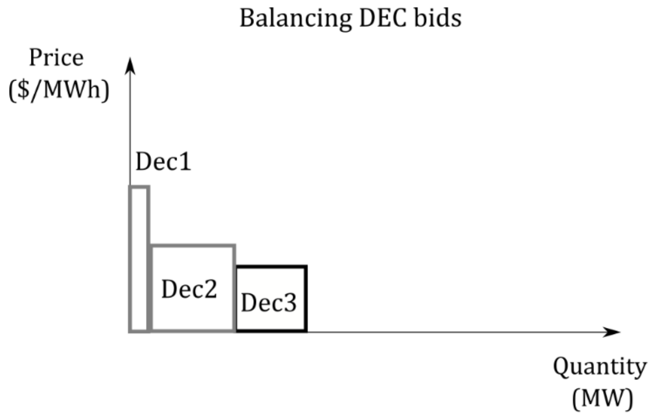
What if the resource would like to correct its position (in the balancing market)?

- Upward change in production/downward change in consumption is paid from the balancing market to the resource → **increment bid**
- Downward change in production/upward change in consumption is paid from the resource to the balancing market → **decrement bid**

Example 6.16: balancing market clearing







- Black bids: consumers
- Gray bids: producers
- Dashed border: inflexible resources
- Solid border: flexible resources that participate in the balancing market
- Lower left: increment bids
- Lower right: decrement bids

Example 6.16 in numbers

Supply offer	Marginal cost ($/MWh)	Quantity (MW)	Flexible?
S1	25	40	No
S2	40	80	Yes
S3	60	80	No
S4	70	50	Yes
S5	75	40	No
S6	100	50	Yes
Demand offer	**Valuation ($/MWh)**	**Quantity (MW)**	**Flexible?**
D1	110	100	Yes
D2	80	120	No
D3	55	90	No
D4	30	70	Yes

Example 6.16 in numbers

Inc offer	Marginal cost ($/MWh)	Quantity (MW)
Inc1	70	30
Inc2	100	50
Inc3	110	100
Dec offer	**Valuation ($/MWh)**	**Quantity (MW)**
Dec1	70	20
Dec2	40	80
Dec3	30	70

Example 6.16 explained

- The first market (e.g. day-ahead market) clears at a price of 70 $/MWh for a quantity of 220 MW

- For the balancing market:
    - Inc1 originates from S4 (flexible resource and for which 30 MW have not been cleared)
    - Inc2 corresponds to bid S6
    - Inc3 corresponds to bid D1
    - Dec1 originates from the 20 MW of bid S4 that have already been cleared
    - Dec2 corresponds to S2
    - Dec3 corresponds to D4

Notation for balancing market model

- D: set of decrement bids
- U: set of increment bids
- MB_d: marginal benefit of decrement bid d
- Δ_d: offered quantity of decrement bid d
- MC_u: marginal cost of increment bid u
- Δ_u: offered quantity of increment bid u
- δ^+ (respectively δ^-): amount of upward (respectively downward) activation that is cleared in the balancing market
- Δ: demand for upward or downward activation (can be positive or negative)

Balancing market model

$$\max_{\delta} \sum_{d \in D} MB_d \cdot \delta_d^- - \sum_{u \in U} MC_u \cdot \delta_u^+$$

$$\sum_{u \in U} \delta_u^+ - \sum_{d \in D} \delta_d^- = \Delta$$

$$\delta_u^+ \leq \Delta_u, u \in U$$

$$\delta_d^- \leq \Delta_d, d \in D$$

$$\delta_u^+ \geq 0, u \in U$$
$$\delta_d^- \geq 0, d \in D$$

Example 6.17: clearing the balancing market

Suppose that the generator offering S3 fails

Using upward offers:

- Shortage of 80 MW (inelastic demand for 80 MW of upward energy)
- The market clearing price is 100 $/MWh (or any price between 100 – 110 $/MWh)
- Offers Inc1 and Inc2 are fully accepted, offer Inc3 is fully rejected

Example 6.17: clearing the balancing market

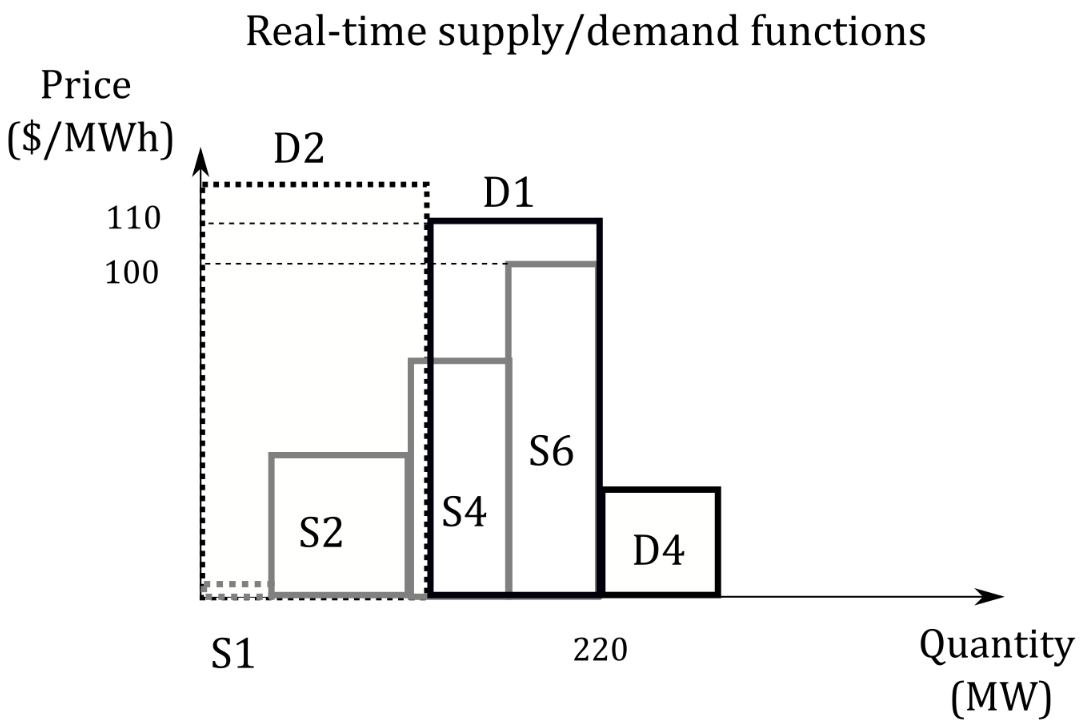
Suppose that the generator offering S3 fails

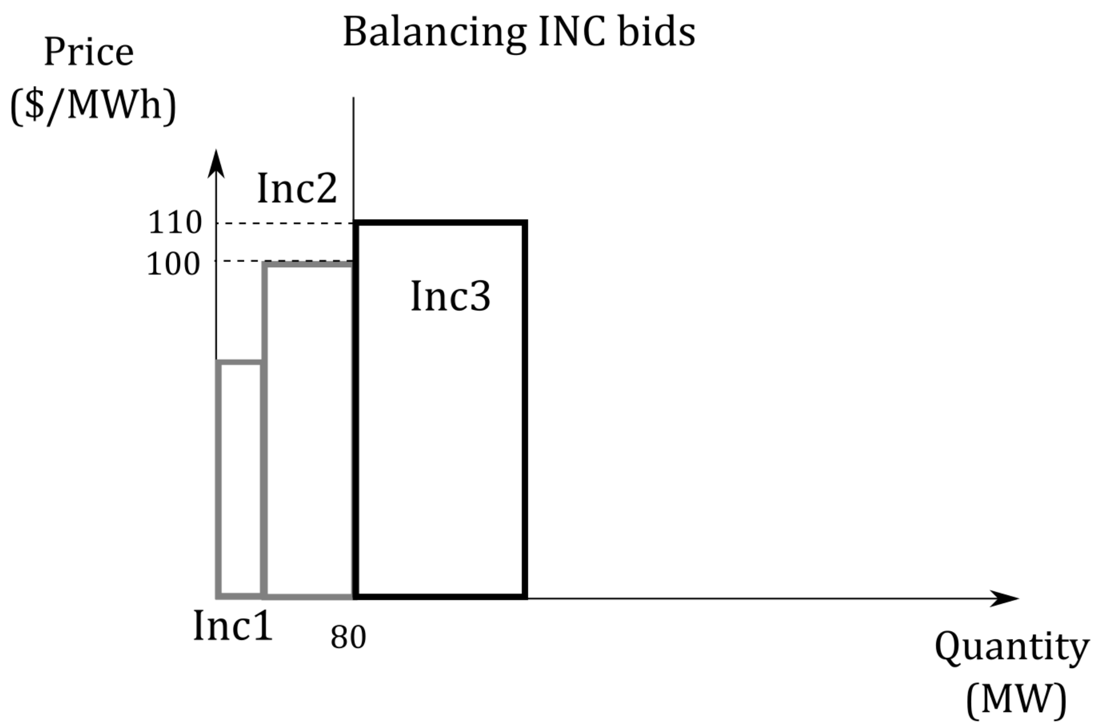
Using economic dispatch:

- Offer S3 is not available because the unit has failed
- Offer S5 is not available because it is not cleared in a preceding market and is not flexible
- Offer D3 is not available because it is not cleared in a preceding market and is not flexible
- Offer S1 shifts to the left of the supply curve because it is inelastic and has been cleared in a preceding market
- Offer D2 shifts to the left of the demand curve because it is inelastic and has been cleared in a preceding market

Important observation: the clearing of upward/downward offers is equivalent to the solution of economic dispatch

Example 6.17: graphical representation





Example 6.17: real-time supply and demand bids

Supply offer	Marginal cost ($/MWh)	Quantity (MW)
S1	25	40
S2	40	80
S4	70	50
S6	100	50
Demand offer	**Valuation ($/MWh)**	**Quantity (MW)**
D1	110	100
D2	80	120
D4	30	70

References

[1] A. Papavasiliou, Optimization Models in Electricity Markets, Cambridge University Press

https://www.cambridge.org/highereducation/books/optimization-models-in-electricity-markets/0D2D36891FB5EB6AAC3A4EFC78A8F1D3#overview

[2] Hogan, William W. “Electricity scarcity pricing through operating reserves.” Economics of Energy & Environmental Policy 2.2 (2013): 65-86